A Fifth Fundamental Force? A Comparative Review Between ΛCDM and Quintessence

Abstract

The nature of dark energy remains one of the most important unsolved problems in cosmology today. The Lambda Cold Dark Matter (ΛCDM) model is currently the most widely accepted model of the Big Bang theory, describing dark energy using a cosmological constant Λ. However, discrepancies arise when attempting to determine its actual value. Along with the Hubble tension, issues like these suggest the need for alternative dark energy models. This review paper primarily focuses on whether ΛCDM is still the most plausible model for dark energy. We first give an overview of observational evidence for dark energy and a review of ΛCDM. Then, we discuss the current issues with ΛCDM. Next, we explore a modified dark energy model — quintessence (of which there are subtypes — freezing and thawing), including its potential functions and equations of state. Finally, we analyze and compare their compatibilities with observational evidence and identify prospects to determine the nature of dark energy. We find that quintessence is preferred over ΛCDM, but observational data also suggest a deviation from quintessence at higher redshifts, thus favoring quintom models (which combine quintessence and phantom behavior), $w_0 w_a$CDM, and $w_0 w_a w_b$CDM over quintessence and ΛCDM.

Keywords

Physics, cosmology, dark energy, quintessence, ΛCDM.

1 Introduction

We can only see about 4% of the universe. That is, only 4% of the universe is made up of atoms (or "ordinary matter"). The rest of the 96% is dark energy (73%) and dark matter (23%) (Coble et al., 2013). Dark matter is a hypothetical form of matter that does not interact with light, but its existence has been inferred from gravitational effects observed at various scales in the universe (Mayet et al., 2016). Many observations suggest that our universe undergoes accelerated expansion Riess et al., 1998), leading to the concept of dark energy — a hypothetical form of energy with negative pressure that causes this expansion — being proposed. Decades of research and observations have been conducted, but the natures of dark matter and dark energy remain a mystery today.

Einstein (1915) developed the Einstein field equations (EFEs), which relate matter and energy in the universe to the geometry of spacetime. The EFEs were a turning point in physics and showed how Newtonian physics was not enough to fully describe the dynamics of astronomical bodies. Einstein (1917) then proposed adding a cosmological constant Λ to the EFEs to obtain a solution for a static and homogeneous universe (which was believed to be the case at the time); this served to counteract the effect of gravitational attraction within the universe. However, Hubble (1929) discovered that the universe was expanding, which led Einstein to remove Λ from the EFEs (Li et al., 2011). This discovery had incredibly profound implications as it meant that there was an unknown form of energy that propelled the universe's expansion. Following this discovery, Sandage (1962) introduced the deceleration parameter q_0, a quantity describing the decelerating rate of expansion of the universe, as it was believed at the time that the universe's expansion was slowing. Zel'Dovich (1968) reintroduced Λ as it denoted the energy density and pressure of a vacuum, accounting for the unknown form of energy driving the universe's expansion. In the following decades, many cosmologists focused on measuring the values of q_0 and Hubble's parameter H, a quantity that describes the ratio between the relative recession velocity of distant celestial bodies and their distances (Hubble, 1929).

Quantum field theory (QFT) is a theory comprising principles from special relativity, quantum mechanics, and classical field theory; it is currently one of the most complete and successful physical theories and is used to formulate the Standard Model of particle physics. Weinberg (1989) calculated that the predicted value of Λ in QFT was around 120 orders of magnitude larger than the observed value. This implied that there was a flaw in QFT or errors in observational methods, indicating a need for further theoretical and empirical development in physics. In 1998 and 1999, observations from various supernovae suggested that the universe's expansion rate was accelerating, supporting the existence of a mysterious form of energy called "dark energy" (Peebles et al., 2003; Perlmutter et al., 1999; Riess et al., 1998), a term coined by Huterer and Turner (1999). Since then, many alternative dark energy and gravity models have been proposed (Abbott et al., 2016; Caldwell et al., 1998; Gu et al., 2009; Liddle et al., 2006; Tutusaus et al., 2024).

There has been ongoing research since the 2000s to find observational evidence to test the validity of different models (Abbott et al., 2016; Gu et al., 2009; Liddle et al., 2006; Tutusaus et al., 2024), including the Dark Energy Survey (DES) and the Dark Energy Spectroscopic Instrument (DESI), which have been collecting data from Type Ia supernovae (SNe Ia) and baryon acoustic oscillations (BAOs), respectively, to further constrain parameters and equations of dark energy models (Abbott et al., 2025; Abdul-Karim et al., 2025; Abdul-Karim et al., 2025). In the 2025 DESI collaboration, it was also found that a dynamical dark energy model is preferred over ΛCDM with a confidence of 2.8–4.2σ (Abdul-Karim et al., 2025). Therefore, the main goal of this paper is to explore the physics of ΛCDM and quintessence, examine their plausibility, and evaluate whether ΛCDM is still the most conceivable model for dark energy. We show that ΛCDM does not remain the most plausible dark energy model and that a dynamical model (e.g., quintessence) best fits observational data.

In this paper, we first review the observational evidence for dark energy, including observed redshift from supernovae and BAOs. Then, we explore the core ideas and issues in ΛCDM that imply beyond-ΛCDM physics. In the subsequent sections, we examine the physics of quintessence — a modified dark energy model that has gained prevalence over the decades due to its relatively simple formulation and better fit to data than ΛCDM (Steinhardt, 2003). We look at the predicted evolution of the equations of state (EoSs) for freezing and thawing quintessence, which, along with the Friedman equations (FEs), determine the evolution of the universe. We aim to provide an overview of the status of quintessence and other modified matter models of dark energy. We also aim to provide a comparative review of the plausibility of quintessence relative to ΛCDM, as a supplement to the existing literature, by comparing predictions to observational data and discussing its strengths and limitations.

We use SI units in this paper. Greek letters such as μ and ν denote 4D indices (0,1,2,3) and Latin letters such as i and j denote 3D spatial indices (1,2,3). Einstein's summation notation (ESN) is assumed throughout. For a review of ESN, see the Appendix.

2 Observational evidence for dark energy

In this section, we provide a brief overview of the two primary pieces of observational evidence supporting the existence of dark energy — observations from Type Ia supernovae and baryon acoustic oscillations.

2.1 Type Ia supernovae

SNe Ia are a type of standard candles (astronomical objects with known luminosities); these supernovae occur in binary star systems when a white dwarf exceeds 1.44 times the mass of the Sun (called the Chandrasekhar mass) and reignites (Chandrasekhar, 1931; Janka et al., 2007). This happens when electron degeneracy pressure can no longer withstand gravity and the star collapses, exploding into a supernova. Because white dwarfs have a fixed Chandrasekhar mass, their peak luminosity is predictable, and by measuring the apparent magnitude, we can deduce their distance from Earth.

Cosmological redshift, similar to the Doppler effect, occurs when a source in the universe that emits a specific wavelength of light becomes more distant from an observer due to expansion, resulting in the observer seeing a longer wavelength of light. However, it is the expansion of space itself that causes the redshift, not the movement of the source itself, as is the case for Doppler redshift. The cosmological redshift z of an object emitting light with wavelength λ_0 and receding with speed v is calculated using the following equation:

$$z = \frac{\lambda - \lambda_0}{\lambda} = \frac{\sqrt{1+\frac{v}{c}}}{\sqrt{1-\frac{v}{c}}}, \#(2.1)$$

where λ is the observed wavelength of light and c is the speed of light. The scale factor $a(t)$ describes the size of the universe relative to its current size. t denotes cosmic time, which is the time elapsed since the Big Bang in a reference frame where the CMB does not appear to have any Doppler shift and the average velocity of matter in the universe is zero. Cosmic time can be thought of as an "absolute" time, taking the universe's expansion into account (Bamonti & Thébault, 2025). $a(t_0) = 1$ denotes the scale factor today and t_0 is the current age of the universe. The scale factor a can be written in terms of redshift z like so:

$$a = \frac{1}{1+z}. \#(2.2)$$

In 1929, Hubble (1929) found that the universe is expanding. Hubble's law states that the recession speed v of a distant galaxy relative to Earth is proportional to the proper distance D between them, given by

$$v = \dot{D} = H_0 D, \#(2.3)$$

where $\dot{D}$ is the derivative of D with respect to t, and H_0 is the Hubble constant (the present-day value of the Hubble parameter H). However, no one knew if the expansion rate was changing. It was believed at the time that the universe's expansion was decelerating, leading to Sandage (1962) introducing the deceleration parameter q_0, given by

$$q_0 = -\frac{\ddot{a}a}{\dot{a}^2}, \#(2.4)$$

where $\dot{a}$ and $\ddot{a}$ denote the first and second derivatives of a with respect to t. Contrary to belief at that time, Riess et al. (1998) observed SNe Ia with redshifts between 0.16 and 0.62 and found that the distances were 10–15% further and dimmer than expected. As a result, a positive Λ with 3.0–4.0σ confidence and a negative deceleration parameter with 2.8–3.9σ confidence were determined. Perlmutter et al. (1999) measured 42 SNe Ia with redshifts between 0.18 and 0.83 and determined with 99% confidence that $\Lambda > 0$. These results imply that the universe is likely undergoing accelerated expansion, with many other observations supporting this (Aghanim et al., 2020; Eisenstein et al., 2005). This means that the recessional velocities of galaxies relative to Earth at higher redshifts increase, and that the Hubble constant H_0 changes over time (hence why we have the Hubble parameter H).

2.2 Baryon acoustic oscillations

Analogous to how SNe Ia are used as standard candles, BAOs are used as standard rulers, which are objects with a known physical size. In the early stages of the universe, photons and matter existed as plasma (where particles are ionized and electrons are unbound to atoms). Where there were higher densities of this plasma, gravity caused it to attract, and pressure caused it to expand, resulting in oscillations with largely fixed wavelengths in the plasma, like sound waves. These oscillations are referred to as BAOs. Afterwards, when the plasma cooled and neutral atoms formed (called the recombination phase), photons were able to travel freely without colliding with electrons frequently, forming the cosmic microwave background (CMB). Over time, the higher-density regions of baryonic matter left behind from the plasma, known as sound horizons, formed stars and galaxies. By measuring the apparent sizes of these sound horizons at different redshifts, we can determine the rate of expansion of the universe (Abbott et al., 2024). Eisenstein et al. (2005) found that the actual size of a sound horizon is around 490 million light-years. These largely fixed separation distances are due to the pressure waves being "frozen" after recombination. Blake et al. (2011) used BAOs to conclude with 99.8% confidence that the universe is undergoing accelerated expansion.

3 ΛCDM and the cosmological constant

In this section, we review the core concepts of ΛCDM. We first look at the EFEs, FEs, and EoS. We then examine the Hubble tension and the cosmological constant problem of ΛCDM. The EFEs, first published by Einstein (1917) relate the mass-energy content (right side of Equation 3.1) in the universe to the curvature of spacetime (left side of Equation 3.1). They are given (in component form) by

$$R_{\mu\nu} - \frac{1}{2}Rg_{\mu\nu} + \Lambda g_{\mu\nu} = \frac{8\pi G}{c^2}T_{\mu\nu}, \#(3.1)$$

where $R_{\mu\nu}$ is the Ricci tensor, R is the Ricci scalar, $g_{\mu\nu}$ is the metric tensor, Λ is the cosmological constant, G is the gravitational constant, c is the speed of light, and $T_{\mu\nu}$ is the stress-energy tensor (which describes the energy and momentum flux in temporal and spatial dimensions). Friedmann (1922) derived the FEs from the EFEs based on the assumptions that the universe — at large scales — is expanding with a time-varying scale factor $a(t)$ for cosmic time t, has uniform energy density $\rho = T_{00}$ (the first component of $T_{\mu\nu}$), and has uniform pressure $p(t) = T_{ii}$ (the diagonal components of $T_{\mu\nu}$), where both ρ and p account for the radiation, matter (including baryonic and dark matter), and dark energy in the universe. That is, the universe is assumed to be a perfect fluid and spatially isotropic and homogeneous. The FEs describe the expansion of the universe, given its mass-energy content and curvature. The first FE is given by

$$H^2 = \left(\frac{\dot{a}}{a}\right)^2 = \frac{8\pi G\rho}{3c^2} - \frac{kc^2}{a^2} + \frac{\Lambda c^2}{3}, \#(3.2a)$$

where $H = \dot{a}/a$ is the Hubble parameter, $\dot{} = \partial/\partial t$ denotes a derivative with respect to cosmic time t, and k is the curvature parameter of the universe, which can be positively curved (+1), flat (0), or negatively curved (−1). The second FE is given by

$$\frac{\ddot{a}}{a} = -\frac{4\pi G}{3}\left(\rho + \frac{3p}{c^2}\right) + \frac{\Lambda c^2}{3}, \#(3.2b)$$

The Friedmann-Lemaître-Robertson-Walker (FLRW) metric — the metric for a spatially isotropic, homogeneous, and flat universe — is derived from the EFEs and FEs. The corresponding line element ds is given by

$$ds^2 = -c^2dt^2 + a(t)^2d\mathbf{x}^2, \#(3.3)$$

where t is cosmic time and $\mathbf{x}$ is a set of 3D spatial coordinates. Here, we use the $(- + + +)$ metric signature. If instead we were to use the $(+ - - -)$ metric signature, the signs of the temporal and spatial terms in Equation 3.3 would switch. For more on the metric tensor, see the Appendix. And by using the FLRW metric, the stress-energy tensor components become $T_{00} = \rho$ and $T_{ii} = a^2p$. The FEs are important because different dark energy models use them to predict different evolutions of the universe (i.e., different Hubble parameters). Another important equation for understanding the dynamics of dark energy is its EoS, given by the ratio of pressure p to energy density ρ for a perfect fluid: $w = p/\rho$. In ΛCDM, the EoS for dark energy equals −1, meaning that the pressure and energy density of dark energy are constant in time and $p = -\rho$. Other components of the universe — including radiation, ordinary matter, and dark matter — have different values of w, so w itself is not unique to dark energy.

While the EFEs and FEs are well-established equations in that the EFEs give extremely accurate predictions of the behavior of astronomical objects and that the FEs are derived from the EFEs (assuming an isotropic and homogeneous universe), huge issues exist in experimentally verifying the value of H, and disagreements arise in different physical theories of Λ.

3.1 Hubble tension

In ΛCDM, dark energy is modeled using a cosmological constant Λ. Astronomers use various methods to determine the present-day value of $H(a)$, called the Hubble constant H_0, with methods including measuring distance using standard candles such as Cepheids (pulsating stars) and SNe Ia, as well as measuring redshifts of galaxies (Riess et al., 1998, 2019). These methods then help determine the scale factor a and solve for H via the FEs. However, a major disagreement exists between measurements of the value of H_0. The 2018 *Planck* collaboration determined a value of H_0 = 67.36 ± 0.54 km s^{-1} Mpc^{-1}, whereas the 2024 SH0ES collaboration measured a value of 73.17 ± 0.86 km s^{-1} Mpc^{-1} at a >5σ tension with the *Planck* collaboration (Zhou et al., 2025). The cause of this discrepancy is unknown and is a huge problem for cosmology as it indicates potential systematic errors in our measurements or a fundamental flaw in ΛCDM, requiring new developments in measurement techniques and mathematical models. Different measurements of H_0 from 2001 to 2022 are shown with error bars in Figure 1.

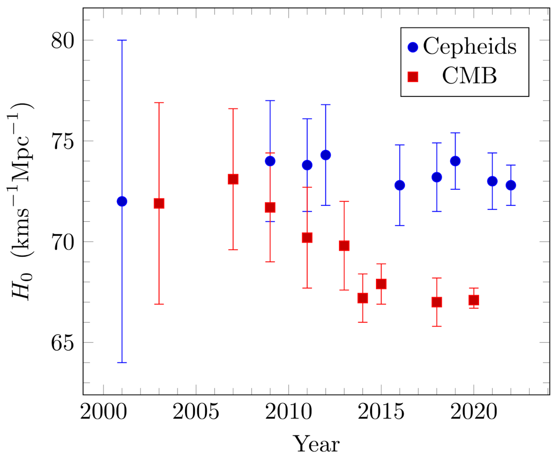

Figure 1. Various measurements of the Hubble constant H_0 from Cepheids (blue) and CMB data (red) from 2001 to 2022. A major discrepancy between the measured value of H_0 exists. Reproduced from (Perivolaropoulos & Skara, 2022).

3.2 Cosmological constant problem

The cosmological constant problem refers to a discrepancy between the observed and predicted values of the vacuum energy density (vacuum energy per cubic meter) $\rho_\Lambda = \Lambda c^4/8\pi G$ (and hence Λ). In QFT, the zero-point energy of a vacuum (also called the vacuum energy) refers to the minimum amount of energy a vacuum can have. Dark energy is a form of vacuum energy where the vacuum energy density is constant, which means that the total amount of dark energy increases as the universe expands. The observed value of ρ_Λ is 2×10^{-11} J m^{-3}, but the predicted value from QFT is 2×10^{109} J m^{-3}, corresponding to a discrepancy of 120 orders of magnitude (Yoo & Watanabe, 2012). This discrepancy is a massive problem as it demonstrates the current incompatibility between theory and cosmological observations; this may require significant reformulation of QFT or theories of cosmology, fundamentally changing our understanding of dark energy.

3.3 Coincidence problem

Current measurements suggest that the universe is extremely flat (i.e., the curvature parameter k is essentially zero). The curvature of the universe is affected by its mass-energy content. The critical density ρ_c is defined as the energy density (of all the content of the universe, including matter, radiation, and dark energy) required for $k = 0$, where we define the ratio ρ/ρ_c as the density parameter Ω. The Wilkinson Microwave Anisotropy Probe (WMAP) and the Supernova Legacy Survey (SNLS) measured $\Omega = 1$ with around 1% deviation (Spergel et al., 2007). Observations also suggest that the density parameter for matter is $\Omega_m \approx 0.3$ and that for dark energy is $\Omega_\Lambda \approx 0.7$. The coincidence problem then asks the question of why Ω_m and Ω_Λ are of the same order of magnitude today despite evolving very differently in the past. Since the universe is measured to be very flat (i.e., $\Omega \approx 1$ and $\rho \approx \rho_c$), this suggests that if ρ_m or ρ_Λ were to slightly deviate away from the critical value, over time, this would result in drastically different evolutions of the universe, including the Big Crunch (where gravity collapses the universe) and the Big Freeze (where the universe grows cold) (Zlatev et al., 1999). Problems such as the Hubble tension, the cosmological constant problem, and the coincidence problem imply the need for new models of dark energy, with one such being quintessence.

4 Quintessence

There are two general types of modified dark energy models. Those that change the left side of the EFEs (Equation 3.1) result in modified gravity models, and those that change the right side result in modified matter models (Tsujikawa, 2013; Yoo & Watanabe, 2012). This paper will focus on quintessence (a term coined by Caldwell et al. (1998)), which is a modified matter model. It is one of the simplest alternatives to Λ involving a change to the stress-energy tensor $T_{\mu\nu}$. Quintessence is postulated as a "fifth fundamental force" that couples gravitationally to matter and dismisses the requirement of Λ, instead modelling dark energy using a dynamic (time-dependent), slowly evolving, and possibly inhomogeneous scalar field $\varphi(x^\mu)$ (though assumed to be homogeneous in this paper, which is an accurate approximation over large scales) that couples only gravitationally to matter, with spacetime coordinates x^μ and a potential energy $V(\varphi)$ that gradually decreases to a minimum (Steinhardt, 2003; Yoo & Watanabe, 2012). This contrasts with ΛCDM, where dark energy remains constant through time.

Quintessence does not solve the Hubble tension nor the cosmological constant problem, but it partially solves the coincidence problem, which arises due to the energy density of dark energy ρ_Λ and matter ρ_m in the universe decreasing at different rates, hence requiring their initial values to be fine-tuned (Zlatev et al., 1999). Quintessence addresses this problem by providing a "tracking" behavior of φ where the quintessence EoS roughly equals the EoS for radiation during the radiation-dominated epoch and that for matter during the matter-dominated epoch of the universe, with the EoS of quintessence only recently becoming dominant. This allows a wide range of initial values for φ to lead to the same evolution of w. However, the potential V must in turn be fine-tuned, which shifts the coincidence problem to "why this potential?".

The stress-energy tensor $T_{\mu\nu}$ for the quintessence field φ is given by (Copeland et al., 2006; Li et al., 2011)

$$T_{\mu\nu} = \partial_\mu\varphi\partial_\nu\varphi - g_{\mu\nu}\left[\frac{1}{2}\partial^\alpha\partial_\alpha\varphi + V(\varphi)\right], \#(4.1)$$

where $g_{\mu\nu}$ is the metric tensor, and $\partial_\mu = \partial/\partial x^\mu$ and $\partial^\mu = \partial/\partial x_\mu$ denote the covariant and contravariant 4-gradients, respectively. We assume pressure and energy density are measured in the local rest frame of an observer comoving with the quintessence field; otherwise, an additional factor of a^2 is included. To get rid of a^2, we raise one index of $T_{\mu\nu}$ with the inverse FLRW metric $g^{\mu\nu}$ = diag($-1,a^{-2},a^{-2},a^{-2}$) such that $T_{\mu\nu}$ = diag(ρ,a^2p,a^2p,a^2p) becomes $T^{\mu}{}_{\nu}$ = diag($-\rho,p,p,p$). By assuming a spatially homogeneous field φ, we see from Equation 4.2 that the energy density $\rho = -T^0{}_0$ and pressure $p = T^i{}_i$ for i = 1, 2, 3 of the field φ in the local rest frame of an observer are given by

$$\rho = T_{00} = \frac{1}{2}\dot{\varphi}^2 + V(\varphi),\ \ p = T_{ii} = \frac{1}{2}\dot{\varphi}^2 - V(\varphi), \#(4.2)$$

leading to the following EoS:

$$w = \frac{p}{\rho} = \frac{\frac{\dot{\varphi}^2}{2} - V(\varphi)}{\frac{\dot{\varphi}^2}{2} + V(\varphi)}. \#(4.3)$$

We see from Equation 4.3 that w can only range from −1 to 1. The EoS can also be written using the Chevallier-Polarski-Linear (CPL) parametrization (Chevallier & Polarski, 2001), a common parametrization found in literature which uses the cosmological scale factor a, w_0 (the present-day value of w with a = 1) and $w_a = -\mathrm{d}w/\mathrm{d}a$ evaluated at a = 1 (corresponding to the negative rate of change of w); it is a first-order Taylor expansion around a = 1 and linearly approximates non-linear solutions to w for high-a. The CPL parametrization is given by (Bayat & Hertzberg, 2025)

$$w(a) = w_0 + w_a(1 - a). \#(4.4)$$

From Equation 4.4, we see that w_0 = 0 and w_a = 1 for ΛCDM. We refer to the cosmological model that assumes cold dark matter (CDM) and the CPL parametrization as the w_0w_aCDM model. For quintessence, since $w \geq -1$, any region of $w < -1$ is called the phantom region (which corresponds to the phantom dark energy model). Thus, the line $w = -1$ is called the phantom divide line (PDL). To compare different dark energy models, we look at the predicted evolution of w with respect to a and z and how it correlates with observational data. There are different classes of quintessence models, with the two main types being freezing (w_a < 0) and thawing (w_a > 0) models. The scalar field φ follows the same equation of motion in both types (Bayat & Hertzberg, 2025; Chevallier & Polarski, 2001; Perivolaropoulos & Skara, 2022; Spergel et al., 2007):

$$\ddot{\varphi} + 3H\dot{\varphi} + V'(\varphi) = 0, \#(4.5)$$

where H is Hubble's parameter and ′ = d/dφ denotes a derivative with respect to φ. The term $3H\dot{\varphi}$ is called Hubble friction since it acts as a dampening term and is what decelerates φ; V′ causes the acceleration of φ. An example freezing model potential is given by

$$V(\varphi) = M^{4+\alpha}\varphi^{-\alpha}, \#(4.6)$$

where M is the mass parameter (chosen so that the quintessence scalar field energy density matches the observed value of the dark energy density ρ_Λ) constrained to be less than or approximately 1.78×10^{-39} kg (Khoury, 2013; Khoury & Weltman, 2004) and $\alpha > 0$ is a constant. An example thawing model potential is given by

$$V(\varphi) = M^4\left[1 + cos\left(\frac{\varphi}{f_a}\right)\right], \#(4.7)$$

where f_a is a constant representing the width of V. The idea behind freezing models is that in the early stages of the universe, $w > -1$ and V had already been decreasing towards a minimum, but as time passes, its rate of decrease slows down and tends towards $w = -1$. Thawing models, on the other hand, have $w \approx -1$ in the early stages and increases with time (Caldwell & Linder, 2005). The EoS of the freezing potential (Equation 4.6) is given by (Chiba et al., 2013; Tsujikawa, 2013)

$$w(a) = w_0 + \sum_{n=1}^{\infty} \frac{(-1)^{n-1} w_0(1-w_0^2)}{1-w_0(n+1)+2nw_0^2(n+1)} \left[\frac{\Omega(a)}{1-\Omega(a)}\right]^n, \#(4.8)$$

where $\Omega(a) = \rho/(3H^{2M_{pl}^2})$ is the field density parameter of φ, ρ is the energy density of φ, H is the Hubble parameter, and M_{pl} is the Planck mass. The 3rd-order analytical solution to $w(a)$ for the freezing potential (Equation 4.8) is plotted in Figure 2.

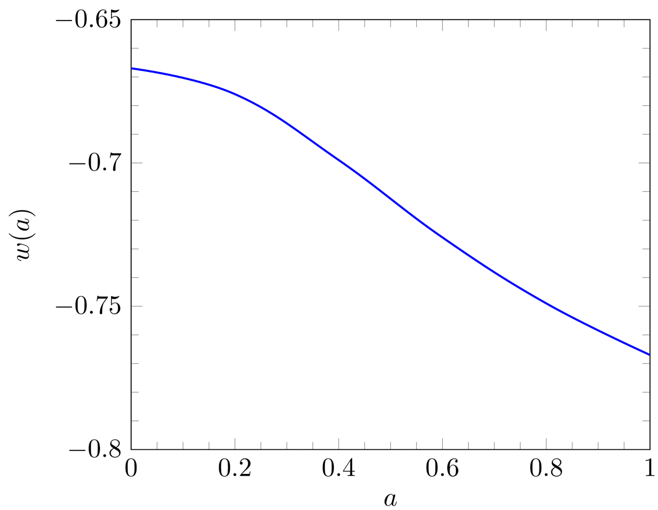

Figure 2. Plot of the 3rd-order analytical solution to $w(a)$ (Equation 4.8) against the scale factor a for the freezing potential (Equation 4.6) with $\alpha = 1$. Reproduced from (Tsujikawa, 2013).

The EoS of the thawing potential (Equation 4.7) is given by

$$w(a) = -1 + \left(1 + w_0\right)a^{3(K-1)}F(a), \#(4.9)$$

where we define the quantities K and $\tilde{F}(a)$ as

$$K \equiv \sqrt{1 - \frac{4M_{pl}^2 V''}{3V}}, \ \tilde{F}(a) \equiv \sqrt{1 + \left(\Omega_0^{-1} - 1\right)a^{-3}}, \#(4.10)$$

such that $F(a)$ is given by

$$F(a) = \left[\frac{(K-\tilde{F})(\tilde{F}+1)^K + (K+\tilde{F})(\tilde{F}-1)^K}{\left(K-\Omega_0^{-\frac{1}{2}}\right)\left(\Omega_0^{-\frac{1}{2}}+1\right)^K + \left(K+\Omega_0^{-\frac{1}{2}}\right)\left(\Omega_0^{-\frac{1}{2}}-1\right)^K}\right], \#(4.11)$$

where $\Omega_0 \approx 0.7$ is the present-day value of Ω for dark energy (Tsujikawa, 2013). The above quantities are arbitrary parameters simply defined to compactify Equation 4.9. Analytical solutions to $w(a)$ for the thawing potential (Equation 4.9) with K = 1.9, 2.9, and 8.2 are plotted in Figure 3.

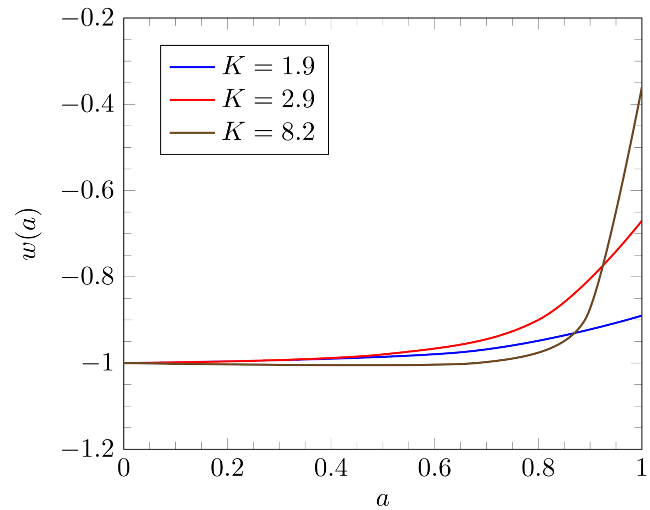

Figure 3. Plot of the analytical solution to $w(a)$ (Equation 4.9) against the scale factor a for the thawing potential (Equation 4.7) with K = 1.9, 2.9, and 8.2. Reproduced from (Tsujikawa, 2013).

5 Results and discussion

In the previous subsection, we discussed the different classes of the quintessence model of dark energy — namely, the freezing and thawing models. In 2025, the Dark Energy Spectroscopic Instrument (DESI) released data from BAO measurements from more than 14 million galaxies and quasars, which were used to constrain various cosmological parameters, including w, w_0, and w_a (Abdul-Karim, Aguilar, et al., 2025). We compare the EoS of freezing and thawing models with CPL constraints from the DESI DR2 data; these are plotted in Figure 4.

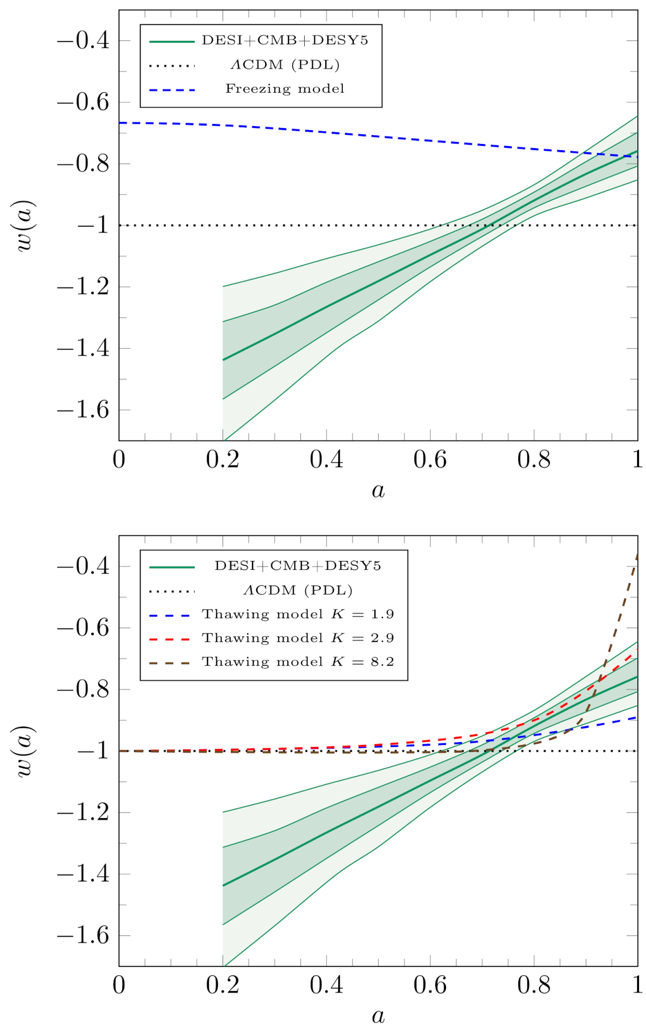

Figure 4. Plots of the EoS $w(a)$ using CPL (green) against scale factor a for freezing (top) and thawing (bottom) quintessence models for $K = 1.9$ (blue), 2.9 (red), and 8.2 (brown) with observational constraints from the 2025 DESI DR2. The dark- and light-green bands around the solid green curve represent a 68% and 95% confidence region for CPL, respectively. The solid green curve represents the observed best fit of w for CPL. Reproduced from (Abdul-Karim, Aguilar, et al., 2025).

From Figure 4, we see that the ΛCDM model falls within the green bands only within around $0.6 < a < 0.75$, but does not deviate as significantly when compared to the freezing model and is thus not completely ruled out based solely on CPL constraints. We see that the freezing model deviates significantly for $a < 0.8$ — well beyond the 95% confidence region when compared to DESI measurements, thus not favored by observations. The thawing model with $K = 2.9$ falls within the 68% and 95% confidence regions for $0.7 < a < 1$, but deviates significantly starting around $a = 0.5$. However, deviations are expected since CPL is only a linear parametrization (in a) and many quintessence potentials lead to a non-linear w; that is, CPL only approximates a local region of a predicted EoS (Linden & Virey, 2008), as seen in Figure 4 with good approximation around $0.6 < w < 1$.

From Figure 4, we also see that at around $a \approx 0.7$, it is likely that w crossed the PDL, corresponding to a transition between quintessence and phantom dark energy models. Phantom dark energy is modelled with a potential ϕ and a negative kinetic energy component; it is characterized by $w \leq -1$. Its EoS is given by

$$w = \frac{p}{\rho} = \frac{\frac{-\dot{\phi}}{2} - V(\phi)}{\frac{-\dot{\phi}}{2} + V(\phi)}. \#(5.1)$$

However, even a purely phantom dark energy model would not satisfy the green bands in Figure 4 for $a > 0.7$. An analysis conducted by Lodha et al. (2025) also showed that freezing and thawing models lack strong support from observational data, indicating that pure quintessence behavior is relatively unfavored by observations. Feng et al. (2005) proposed a quintom model (a portmanteau of quintessence and phantom) where w was allowed to transition between quintessence and phantom behavior.

Higher-order parametrizations based on CPL have also been proposed and analyzed to better approximate dark energy models (Nesseris et al., 2025). An extended parametrization based on CPL is given by

$$w(a) = w_0 + w_a(1 - a) + w_b(1 - a)^2, \#(5.2)$$

where w_0 is the present-day value of w, $w_a = -dw/da$ at $a = 1$, and $w_b = -(d^2w/da^2)/2$ at $a = 1$. Models assuming CDM and the extended CPL parametrization given by Equation 5.2 are dubbed $w_0w_aw_b$CDM. Figure 5 compares freezing and thawing quintessence EoSs with $w_0w_aw_b$CDM constraints from the 2024 DESI DR1.

The DESI collaboration also provided measurements of the possible ranges for w_0 and w_a for the CPL parametrization of w (Abdul-Karim, Aguilar, et al., 2025), shown in Figure 6. We see that the point with $w_0 = -1$ and $w_a = 0$, corresponding to ΛCDM, falls outside the 68% and 95% confidence regions, with the DESI data rejecting ΛCDM with confidence 2.8–4.2σ. We also see that the DESI data provides relatively consistent results favoring $w_0 > 1$ and $w_a < 0$, further indicating deviation from ΛCDM. Wolf et al. (2024) showed that there is minimal overlap between the predicted and observed regions for w_0 and w_a for thawing models. However, their analysis used the potential $V(\varphi) = V_0 + m^2\varphi^2/2$, which they argued represents various potentials and covers a wide range of values for w_0 and w_a.

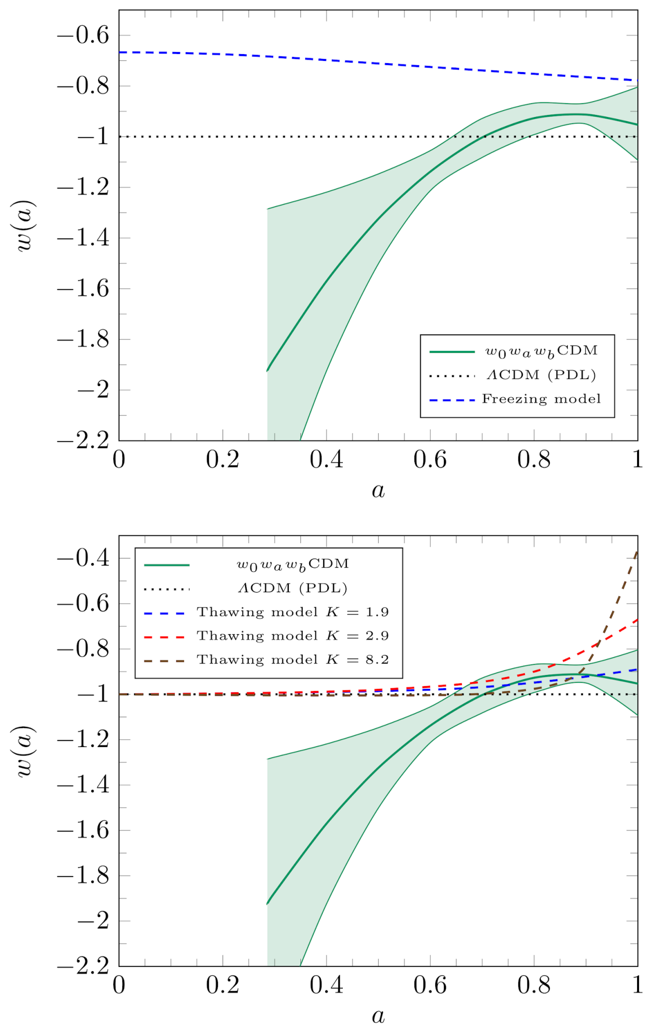

Figure 5. Plots of the EoS $w(a)$ using extended CPL parametrization against scale factor a for freezing (top) and thawing (bottom) quintessence models with observational constraints from the 2024 DESI DR1. The green band around the solid green curve represents a 68% confidence region for extended CPL. The solid green curve represents the observed best fit of w under extended CPL parametrization. Reproduced from (Abdul-Karim, Aguilar, et al., 2025) and (Nesseris et al., 2025).

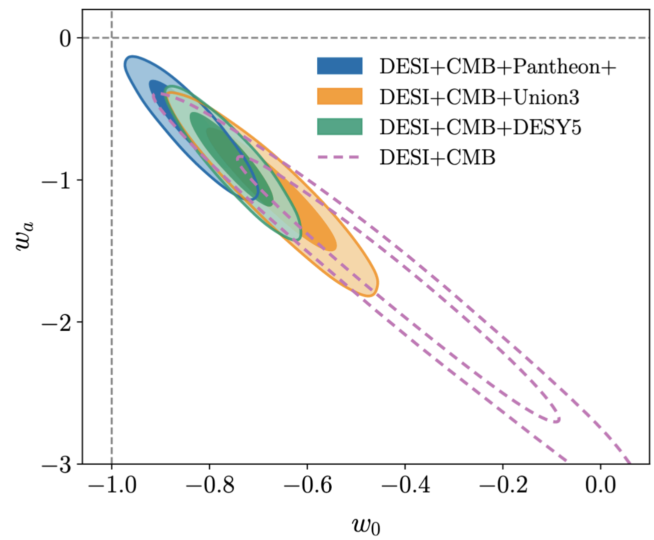

Figure 6. Contour plots of the posterior distributions of w_0 and w_a, with the darker and lighter regions (or the smaller and larger curves) representing a 68% and 95% confidence level, respectively. The intersection of the grey dashed lines represents the values of w_0 and w_a for ΛCDM. Image from (Abdul-Karim, Aguilar, et al., 2025).

Lodha et al. (2025) conducted further analysis of the DESI DR2 data to test the consistency of dynamic dark energy models without a functional form of w. They compared various parametrizations, including the CPL, Barboza-Alcaniz (BA), exponential (EXP), logarithmic (LOG), and Jassal-Bagla-Padmanabhan (JBP) parametrizations, also including an EoS written in terms of Chebyshev polynomials. They found that the fitting of data and plausibility of dynamical dark energy models had little dependence on parametrization, which is strong evidence for the implausibility of ΛCDM for dark energy. Lodha et al. also used a binning approach, an unbiased method of comparing w at different redshift intervals without assuming a functional form of w. This ensures that we can reconstruct the evolution of the universe's expansion (e.g., through w and the FEs) with minimal presumptions about the specific model of dark energy. Figures 7 and 8 compare the data from binning and constraints for w_0w_aCDM and $w_0w_aw_b$CDM, respectively. These results suggest evidence for a quintom scenario (i.e., w_0w_aCDM, $w_0w_aw_b$CDM, and the like), with $w_0w_aw_b$CDM appearing to fit the binning data slightly better than w_0w_aCDM (although with more uncertainty at low-a regions). However, we must keep in mind that alternative parametrizations or models are still not ruled out — including ΛCDM — since current observations are unable to measure w precisely for non-local (smaller) values of a. CPL and extended CPL are also only Taylor approximations about $a = 1$, so higher-order behavior in a of w potentially cannot be captured by these parametrizations.

In the analysis of the DESI data by Gialamas et al. (2025), however, it was argued that although phantom behavior is suggested by observational data, it is not statistically robust as pure quintessence behavior with $w \geq -1$ is still supported with 2σ confidence. In their analysis, they included a Higgs-like quintessence (HLQ) model without phantom behavior; despite CPL matching observational data slightly better than HLQ, HLQ is motivated through physical theory, unlike CPL, which was developed only to parametrize and fit observational data.

The reason for varied predicted evolutions of w from different dark energy parametrizations and models is due to degeneracy, which is the idea that many different models of dark energy can give rise to the same cosmological observations, and our current observational data is not sensitive enough to distinguish the different scenarios for w at high-z (low-a) (Marttens et al., 2020). Nevertheless, we conclude with fairly high confidence that the current data favors dynamical dark energy over ΛCDM.

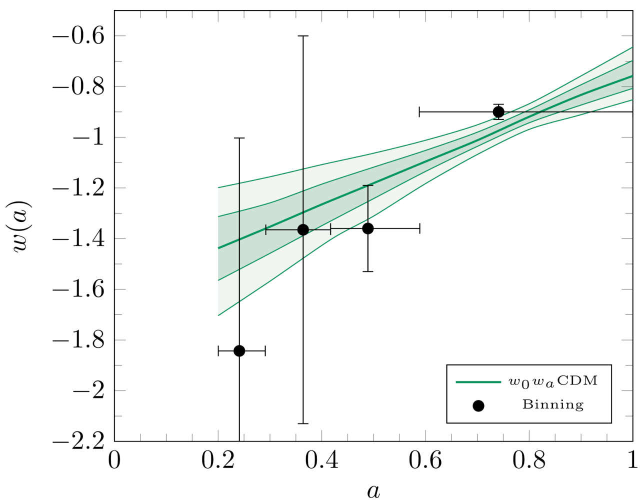

Figure 7. A comparison between w_0w_aCDM constraints (with the dark- and light-green contours representing 68% and 95% confidence regions) and data from the binning method. The black bins represent a 1σ error. Reproduced from (Abdul-Karim, Aguilar, et al., 2025).

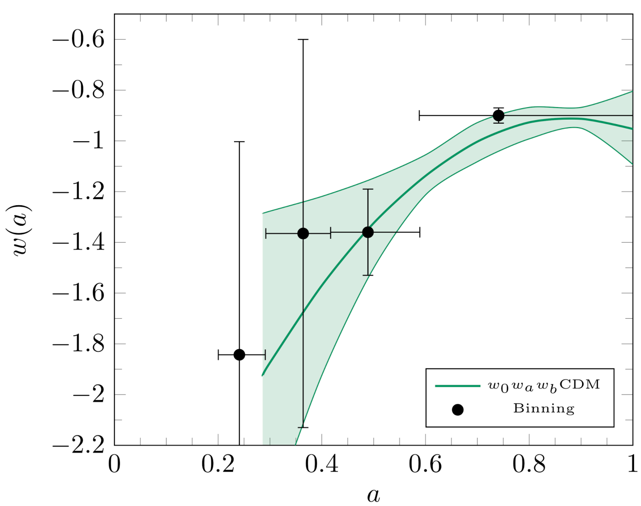

Figure 8. A comparison between $w_0w_aw_b$CDM constraints (with the dark-green contours representing a 68% confidence region) and data from the binning method. The black bins represent a 1σ error. Reproduced from (Abdul-Karim, Aguilar, et al., 2025) and (Nesseris et al., 2025).

6 Conclusion

In this paper, we discussed the main pieces of evidence supporting the accelerated expansion of the universe and motivated the development of alternative dark energy theories by discussing current issues with ΛCDM. We then reviewed the core concepts of ΛCDM, including the EFEs, FEs, and the Λ model for dark energy, and explored the basic physics of quintessence. We compared the predicted EoS from quintessence to observational data from the DESI collaboration and found that quintessence is a more preferred model than ΛCDM. However, due to strong evidence for a phantom crossing, we conclude that quintessence nor phantom fully match observational data. We instead find that models including w_0w_aCDM and $w_0w_aw_b$CDM fit observational data better, with dynamical dark energy models being more favorable than constant ones.

However, this paper is not a comprehensive review of all dark energy models; we only focused on particular modified matter models (with an emphasis on quintessence) and how closely their predictions fit observational data. Hence, further comparisons and research into other modified matter models (such as phantom, K-essence, Chaplygin gas, and chameleon) and modified gravity models (such as $f(R)$ gravity, modified Newtonian dynamics, and DGP braneworld) (Yoo & Watanabe, 2012) are needed. This paper also mainly focused on comparing predicted evolutions of the EoS for each model; there are, however, other methods of assessing the plausibility of a model, such as its mathematical complexity and the number of parameters required to describe the model. We defer further analysis of dark energy models, especially of phantom and quintom, to future work.

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Appendix

Einstein's summation notation (ESN) is a convenient way of denoting sums of indexed terms. Consider a vector $\boldsymbol{v}$, which can be expressed as a linear combination of n basis vectors $\boldsymbol{e}_i$ with n corresponding vector components v^i (where i is the index denoting the i-th object). Upper and lower indices denote contravariance and covariance with respect to basis vectors (so a covector $\boldsymbol{\alpha} = \alpha_i \boldsymbol{\varepsilon}^i$ has covariant components α_i and contravariant basis covectors $\boldsymbol{\varepsilon}^i$). ESN simplifies the notation in such a way that the usual "capital sigma" symbol in front of a sum is omitted:

$$v = \sum_{i=1}^{n} v^i e_i = v^i e_i. \#(A.1)$$

The metric tensor $\boldsymbol{g}(\cdot,\cdot)$ is a bilinear form that takes in two vectors (from the same vector space) $\boldsymbol{u} = u^i \boldsymbol{e}_i$ and $\boldsymbol{v} = v^j \boldsymbol{e}_j$ and outputs a number:

$$g(u, v) = g_{ij} u^i v^j, \#(A.2)$$

where $g_{ij} = \boldsymbol{e}_i \cdot \boldsymbol{e}_j$ are the covariant components of the metric tensor, which tell us about how the local geometry of a manifold changes. One can also think of the metric tensor as a "bar scale" at each point in a manifold, indicating how a local patch of geometry is stretched or squished and how measurements of length change. For example, consider the metric tensor for an n-dimensional Euclidean space (which is a flat space). Its metric tensor is one where all its diagonal components are 1 and the rest are 0: $g_{ij} = \mathrm{diag}(1,\ldots,1)$, where $i, j = 1, 2, \ldots, n$. The metric tensor for Minkowski space (flat 4D spacetime) is given by $\eta_{\mu\nu} = \mathrm{diag}(-1,1,1,1)$ or $\eta_{\mu\nu} = \mathrm{diag}\,(1,-1,-1,-1)$ depending on the convention used, where $\mu, \nu = 0, 1, 2, 3$. This is the metric tensor for the local rest frame of an observer at a point (since an observer always appears at rest with respect to itself, which implies a locally flat spacetime). The inverse metric $\boldsymbol{g}^{-1}$ has contravariant components g^{ij} defined by

$$g^{ik} g_{kj} = \delta^i_j, \#(A.3)$$

where $\delta^i_j = \mathrm{diag}(1,1,1,1)$ is the Kronecker delta (in 4D).

Author 100063 - Submission 100058 - Initial Submission Review

The paper has many strong features, including the clear question under consideration, the data provided to support the conclusions as well as the structure provided. Any reader would have an easy time reading the paper, since the paper nicely expands on the topic and includes detailed and well explained background theories. What is more, the paper's sections follow a logical order, and are all properly connected.

Even though there is no data analysis by the student itself, the paper supports its main claims with the current/existing literature. Many papers used throughout this work, are from recent studies, showing the up-to-date relevance of the paper. The paper helps summarising how the new observational data/constraints allow for new models of Dark Energy to be discussed/promoted.

The author would still benefit from some suggestions, even though the paper doesn't lack clarity. Suggestions (as listed below) are mostly related to figures and equations. Overall, the author seems confident with the level of mathematics used in the paper and makes a good use of the appendix in the end to explain mathematical notation not as common to high school students. Moreover, the author communicates clearly the ideas under consideration with the data provided backing up the author's claims.

When scanning the paper for plagiarism and AI, I get high percentages for AI detection (using turbo search). I suspect this could be a false positive, especially if the student used tools such as Grammarly. I would suggest that the team double checks this.

Finally, I would gladly accept this paper for publication with only minor revisions. The table below outlines suggestions that could help better this paper.

Line Number	**Quote from paper**	**Suggestions/Comments**
9-14	We first give an overview of observational evidence for dark energy and a review of ΛCDM. Then…	The abstract is very good, but using passive voice sometimes helps with the tone of the paper. For example, instead of: we discuss the current issues with ΛCDM, you can say – current issues with ΛCDM are discussed etc.
24	undergoes accelerated expansion Riess et al., 1998),	Typo: (missing
70	of state (EoSs)	Is it (Eos) instead?
154	, $\dot{}$ = $\partial/\partial t$ denotes a derivative with respect..	Could be added in the appendix since ($\dot{}$) is also mentioned earlier?
162-163	Here, we use the ([illegible] + + +) metric signature. If instead we were to..	The squares should change to (-)
179	the present-day value of H(a), called	Should it be H(t)?

212-213	density parameter for matter is Ωm ≈ 0.3 and that for dark energy is ΩΛ ≈ 0.7. The coincidence problem then asks the question of why Ωm and ΩΛ are of the same order of	Font error
241-242	However, the potential *V* must in turn be fine-tuned, which shifts the coincidence problem to "why this potential?".	Can we expand as to why we then choose quintessence, if still fine-tuning is required?
253-255	Derivation of equations	Readers could benefit from some workings. (maybe in the appendix?)
265	Any region of	Font error
273-274	2=d/dφ and V2	Does student mean V' ?
279	And α>0 is a constant	Could be another letter since (a) is used for cosmic scale factor and quite similar to (α).
285	In the early stages and increase..	Font error
295-299	Equations	I believe it would have been better if $\tilde{F}(a)$ switches order with F(α).
300	$\Omega_0 \approx 0.7$ is the present-day value of	Isn't it $\Omega\ \Lambda$? Also, font error.
290 and 303	Figure 2 and 3.	Could add a subscript to w(a) to separate the freezing with the thawing potential.
325-326	only a linear parametrization (in *a*) and many quintessence potentials lead to a non-linear *w*	Nice to offer explanation for deviation.
328	From figure 4..	I would have preferred to see subfigures such as 4a and 4b so that the author can point to them directly.
328	It is likely	Font error.
331	Its EoS is given by	Font error.
332	Equation 5.1	Can just mention equation 4.3
385	Is still supported	Font error
387	HLQ is motivated through physical theory	Is there a paper for this? Can we see a reference?

A general suggestion would also be to number the pages of the paper and to avoid the # that indicates the equation number. For example:

$$a = \frac{1}{1+z}.\#(2.2)$$

Can be written as:

$$a = \frac{1}{1+z} \qquad (2.2)$$

Review of "A Fifth Fundamental Force? A Comparative Review Between ΛCDM and Quintessence"

This review paper, written by a high school student, is an impressive attempt to tackle one of the most complex problems in modern cosmology: the nature of dark energy. First and foremost, I want to recognize the ambition and effort involved here. Engaging with Einstein's field equations, Friedmann models, quintessence potentials, and even the latest DESI observational data is far from trivial, and the author shows both enthusiasm and persistence in learning advanced material. For someone at the high school level, this is an outstanding achievement.

Strengths: The paper is well-structured and demonstrates a clear attempt to walk the reader through the history of dark energy research, from Einstein's cosmological constant to modern dynamical models. The sections on observational evidence (supernovae and BAOs) are explained in a way that shows good understanding of the underlying concepts. The author also demonstrates awareness of current debates, such as the Hubble tension and the cosmological constant problem, which shows they are engaging with cutting-edge questions. The inclusion of equations and plots reflects a serious effort to connect theory and data, which is remarkable for a high school researcher.

Areas for Improvement: Because this is written at an early stage in the author's academic journey, there are natural areas where improvement is needed. The biggest issue is that the paper sometimes feels more like a compilation of textbook material than a critical review. At this level, it is absolutely fine to rely heavily on summarizing existing literature, but the next step is to move from "describing" models to "evaluating" them — asking, for example, *why* quintessence may be more plausible than ΛCDM in practice, and *what the weaknesses* of each model are.

The explanations of the freezing and thawing quintessence models, for example, are mathematically correct, but the physical meaning of these equations is not always made clear in plain language. A good exercise for revision would be to imagine explaining these models to another high school student: what is the simplest way to capture why they matter?

Finally, the writing could be streamlined. At times the paper becomes long and dense, with many technical details that risk overwhelming readers. Trimming repetition and adding more plain-language explanations would make the argument more accessible and pedagogically stronger.

Recommendation: Overall, this paper shows excellent initiative and intellectual curiosity, and I want to strongly encourage the author to keep pursuing cosmology at higher levels of study. For publication, however, the paper would benefit from **major revisions**: clearer explanations in plain language, more critical evaluation of the models, and tighter organization.

I would therefore recommend **"Accept with major revisions".** With further work, this could become a very strong student contribution that not only summarizes technical material but also helps other learners understand the big picture of dark energy research.

A Fifth Fundamental Force? A Comparative Review Between ΛCDM and Quintessence

Abstract

The nature of dark energy remains one of the most important unsolved problems in cosmology today. The Lambda Cold Dark Matter (ΛCDM) model is currently the most widely accepted model of the Big Bang theory, describing dark energy using a cosmological constant Λ. However, discrepancies arise when attempting to determine its actual value, and predictions from theories differ from observations; . Along with the Hubble tension, issues like these these suggest the need for alternative dark energy models. This review paper primarily focuses on whether ΛCDM is still the most plausible model for dark energy and if quintessence is a suitable alternative. We first give anThis paper first gives an overview of observational evidence for dark energy and a review of ΛCDM. Then, we discuss the current issues with ΛCDM are discussed. Next, we explore a modified dark energy model, — quintessence, (of which there are subtypes — freezing and thawing) is explored, including its potential functions and equations of state. Finally, we analyze and comparethis paper analyzes and compares their compatibilities with observational evidence and identify prospects to determine the nature of dark energy. We It is concluded find that quintessence is generally preferred over ΛCDM, but observational data also suggest a deviation from quintessence at higher redshifts, thus slightly favoring quintom models (which combine quintessence and phantom behavior), w_0w_aCDM, and $w_0w_aw_b$CDM over quintessence and ΛCDM. Current observational data are also limited in terms of their ability to meaningfully constrain dark energy parameters.

Keywords

Physics, cosmology, dark energy, quintessence, ΛCDM.

1 Introduction

We can only see about 4% of the universe. That is, only 4% of the universe is made up of atoms (or “ordinary matter”), and t. The rest of the 96% is dark energy (73%) and dark matter (23%) (Coble et al., 2013). Dark matter is a hypothetical form of matter that does not interact with light, but its existence has been inferred from gravitational effects observed at various scales in the universe (Mayet et al., 2016). Many observations suggest that our universe undergoes accelerated expansion (Riess et al., 1998), leading to the concept ofproposal of dark energy — a hypothetical form of energy with negative pressure that causes this expansion — being proposed. Decades of research and observations have been conducted, but the natures of dark matter and dark energy remain a mystery today.

Einstein (1915) developed the Einstein field equations (EFEs), which relate matter and energy in the universe to the geometry of spacetime. The EFEs were a turning point in physics, and showinged how Newtonian physics was not enough to fully describe the dynamics of astronomical bodies. Einstein (1917) then proposed adding a cosmological constant Λ to the EFEs to obtain a solution for a static and homogeneous universe (which was believed to be the case at the time); this served to counteract the effect of gravitational attraction within the universe. However, Hubble (1929) discovered that the universe was expanding, leading~~, which led~~ Einstein to ~~remove~~ remove Λ from the EFEs (Li et al., 2011). This discovery had ~~incredibly~~ profound implications as it meant that there was an unknown ~~form of energy~~component of the universe ~~that~~ propell~~ing~~ed ~~the universe'~~its expansion; hence, Λ was reintroduced by Zel'Dovich (1968). Following this discovery, Sandage (1962) introduced the deceleration parameter q_0, a quantity describing the decelerating rate of expansion of the universe, as it was believed ~~at the time~~ that the universe's expansion was slowing~~. Zel'Dovich (1968) reintroduced Λ as it denoted the energy density and pressure of a vacuum, accounting for the unknown form of energy driving the universe's expansion~~.~~ ~~ In the following decades, many cosmologists focused on measuring the values of q_0 and Hubble's parameter H, a quantity that describes the ratio between the relative recession velocity of distant celestial bodies and their distances (Hubble, 1929). In~~.~~

~~Quantum field theory (QFT) is a theory comprising principles from special relativity, quantum mechanics, and classical field theory; it is currently one of the most complete and successful physical theories and is used to formulate the Standard Model of particle physics. Weinberg (1989) calculated that the predicted value of Λ in QFT was around 120 orders of magnitude larger than the observed value. This implied that there was a flaw in QFT or errors in observational methods, indicating a need for further theoretical and empirical development in physics. In~~ 1998 and 1999, observations ~~, observations~~ from various supernovae ~~suggested~~ supported ~~that~~ the idea that the universe's expansion rate was accelerating ~~, supporting the existence of a mysterious form of energy called "dark energy"~~ (Peebles et al., 2003; Perlmutter et al., 1999; Riess et al., 1998). ~~, a term coined by Huterer and Turner (1999).~~ Since then, many alternative dark energy and gravity models have been proposed (Abbott et al., 2016; Caldwell et al., 1998; Gu et al., 2009; Liddle et al., 2006; Tutusaus et al., 2024), and various observations have been made to test the validity of different models (Abbott et al., 2016; Gu et al., 2009; Liddle et al., 2006; Tutusaus et al., 2024).

~~There has been ongoing research since the 2000s to find observational evidence to test the validity of different models (Abbott et al., 2016; Gu et al., 2009; Liddle et al., 2006; Tutusaus et al., 2024), including the Dark Energy Survey (DES) and the Dark Energy Spectroscopic Instrument (DESI), which have been collecting data from Type Ia supernovae (SNe Ia) and baryon acoustic oscillations (BAOs), respectively, to further constrain parameters and equations of dark energy models (Abbott et al., 2025; Abdul-Karim et al., 2025; Abdul-Karim et al., 2025). In the 2025 DESI collaboration, it was also found that a dynamical dark energy model is preferred over ΛCDM with a confidence of 2.8–4.2σ (Abdul-Karim et al., 2025). Therefore, the main goal of this paper is to explore the physics of ΛCDM and quintessence, examine their plausibility, and evaluate whether ΛCDM is still the most conceivable model for dark energy. We show that ΛCDM does not remain the most plausible dark energy model and that a dynamical model (e.g., quintessence) best fits observational data.~~

~~In this paper,~~This paper ~~we~~ first reviews the observational evidence for dark energy, including observed redshift from supernovae and BAOs. Then, ~~we explore~~ the core ideas and issues in ΛCDM that imply beyond-ΛCDM physics~~.~~ are explored. ~~In the~~The subsequent sections~~,~~ ~~we~~ examine the physics of quintessence, ~~—~~a modified dark energy model that has gained prevalence over the decades due to its relatively simple formulation and better fit to data than ΛCDM (Steinhardt, 2003), and evaluate whether ΛCDM is still the most conceivable model for dark energy. ~~We look~~ This paper looks at the predicted evolution of the equations of state (EoS~~s~~) for freezing and thawing quintessence, which, along with the Friedman equations (FEs), determine the evolution of the universe. ~~We aim to~~

~~provide an overview of the status of quintessence and other modified matter models of dark energy.~~ ~~We~~ This paper also aims to provide a comparative review of the plausibility of quintessence relative to ΛCDM, as a supplement to the existing literature, by comparing predictions to observational data and discussing its strengths and limitations. This paper shows that ΛCDM does not remain the most plausible dark energy model and that a dynamical model better fits observational data.

~~We use~~ SI units are used in this paper. ~~Greek letters such as μ and ν denote 4D indices (0,1,2,3) and Latin letters such as i and j denote 3D spatial indices (1,2,3).~~ Einstein's summation notation (ESN) is assumed throughout. For a review of ESN, see the Appendix.

2 Observational evidence for dark energy

~~In t~~This section~~, we~~ provides a brief overview of the two primary pieces of observational evidence supporting the existence of dark energy — observations from Type Ia supernovae and baryon acoustic oscillations.

2.1 Type Ia supernovae

SNe Ia are a type of standard candles (astronomical objects with known luminosities); these supernovae occur in binary star systems when a white dwarf exceeds 1.44 times the mass of the Sun (called the Chandrasekhar mass) and reignites (Chandrasekhar, 1931; Janka et al., 2007). This happens when electron degeneracy pressure can no longer withstand gravity and the star collapses, exploding into a supernova. Because white dwarfs have a fixed Chandrasekhar mass, their peak luminosity is predictable, and by measuring the apparent magnitude, ~~we~~ one can deduce their distance from Earth.

Cosmological redshift~~, similar to the Doppler effect,~~ occurs when a source in the universe that emits a specific wavelength of light becomes more distant from an observer due to the expansion of space itself, resulting in the observer seeing a longer wavelength of light. ~~However, it is the expansion of space itself that causes the redshift, not the movement of the source itself, as is the case for Doppler redshift.~~ The cosmological redshift z of an object emitting light with wavelength λ_0 and receding with speed v is calculated using the following equation:

$$z = \frac{\lambda\lambda - \lambda_0\lambda_0}{\lambda_0} = \frac{\sqrt{1+v/c}}{\sqrt{1-v/c}}, \qquad (2.1)$$

where λ is the observed wavelength of light ~~and c is the speed of light~~. ~~The~~ Another related quantity is the scale factor $a(t)$, which describes the size of the universe relative to its current size. t denotes cosmic time, which is the time elapsed since the Big Bang in a reference frame where the CMB does not appear to have any ~~Doppler~~ redshift and the average velocity of matter in the universe is zero. Cosmic time can be thought of as an "absolute" time, taking the universe's expansion into account (Bamonti & Thébault, 2025). $a(t_0) = 1$ denotes the scale factor ~~today and t_0 is the~~at the current age of the universe t_0. The scale factor a as a function of z is given by~~can be written in terms of redshift z like so~~:

$$a(z) = \frac{1}{1+z}. \qquad (2.2)$$

~~In 1929, Hubble (1929) found that the universe is expanding. Hubble's law states that the recession speed v of a distant galaxy relative to Earth is proportional to the proper distance D between them, given by~~

$$v = \dot{D} = H_0 D, \qquad (2.3)$$

~~where $\dot{D}$ is the derivative of D with respect to t, and H_0 is the Hubble constant (the present-day value of the Hubble parameter H). However, no one knew if the expansion rate was changing. It was believed at the time that the universe's expansion was decelerating, leading to Sandage (1962) introducing the deceleration parameter q_0, given by~~

$$q_0 = -\frac{\ddot{a}a}{\dot{a}^2}, \qquad (2.4)$$

~~where $\dot{a}$ and $\ddot{a}$ denote the first and second derivatives of a with respect to t. Contrary to belief at that time,~~ Riess et al. (1998) observed SNe Ia with redshifts between 0.16 and 0.62 and found that the distances were 10–15% further and dimmer than expected. As a result, a positive Λ with 3.0–4.0σ confidence and a negative deceleration parameter with 2.8–3.9σ confidence were determined. Perlmutter et al. (1999) measured 42 SNe Ia with redshifts between 0.18 and 0.83 and determined with 99% confidence that $\Lambda > 0$. These results imply that the universe is likely undergoing accelerated expansion, with many other observations supporting this (Aghanim et al., 2020; Eisenstein et al., 2005). This means that the recessional velocities of galaxies relative to Earth at higher redshifts increase, and that the Hubble constant H_0 changes over time ~~(hence why we have the Hubble parameter H)~~.

2.2 Baryon acoustic oscillations

Analogous to how SNe Ia are used as standard candles, BAOs are used as standard rulers, which are objects with a known physical size. In the early stages of the universe, photons and matter existed as plasma (where particles are ionized and electrons are unbound to atoms). Where there were higher densities of this plasma, gravity caused it to attract, and pressure caused it to expand, resulting in oscillations with largely fixed wavelengths in the plasma, like sound waves. These oscillations are referred to as BAOs. Afterwards, when the plasma cooled and neutral atoms formed (called the recombination phase), photons were able to travel freely without colliding with electrons frequently, forming the cosmic microwave background (CMB). Over time, the higher-density regions of baryonic matter left behind from the plasma, known as sound horizons, formed stars and galaxies. By measuring the apparent sizes of these sound horizons at different redshifts, ~~we~~ one can determine the rate of expansion of the universe (Abbott et al., 2024). Eisenstein et al. (2005) found that the actual size of a sound horizon is around 490 million light-years. These largely fixed separation distances are due to the pressure waves being "frozen" after recombination. Blake et al. (2011) used BAOs to conclude with 99.8% confidence that the universe is undergoing accelerated expansion.

3 ΛCDM and the cosmological constant

~~In t~~This section~~, we~~ reviews the core concepts of ΛCDM. ~~We first look at t~~The EFEs, FEs, and EoS are discussed first. ~~We t~~Then, ~~examine~~ the Hubble tension and the cosmological constant problem of ΛCDM are examined. The EFEs ~~, first published by Einstein (1917)~~ relate the mass-energy content (right

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side of Equation 3.1) in the universe to the curvature of spacetime (left side of Equation 3.1). They are given (in component form) by

$$R_{\mu\nu} - \frac{1}{2}Rg_{\mu\nu} + \Lambda g_{\mu\nu} = \frac{8\pi G}{c^2}T_{\mu\nu}, \tag{3.1}$$

where $R_{\mu\nu}$ ~~is the Ricci tensor~~(the Ricci tensor) and R (the Ricci scalar) describe the local curvature of spacetime)~~, R is the Ricci scalar~~, $g_{\mu\nu}$ is the metric tensor (which one can think of as a "bar scale" at each point in a manifold, indicating how a local patch of geometry is stretched or squished and how measurements of length change ~~For more on the metric tensor, see the Appendix.~~), Λ is the cosmological constant, G is the gravitational constant, c is the speed of light, and $T_{\mu\nu}$ is the stress-energy tensor (which describes the energy and momentum flux in temporal and spatial dimensions). Friedmann (1922) derived the FEs from the EFEs based on the assumptions that the universe — at large scales — is expanding with a time-varying scale factor $a(t)$ for cosmic time t, has uniform energy density $\rho = T_{00}$ (the ~~first~~ zeroth temporal component of $T_{\mu\nu}$), and has uniform pressure a^2p~~(t)~~ = $a^2(p_x + p_y + p_z)$ = T_{ii} (the sum of the diagonal spatial components of $T_{\mu\nu}$), where both ρ and p account for the radiation, baryonic and dark matter, ~~(including baryonic and dark matter),~~ and dark energy in the universe. That is, the universe is assumed to be a perfect fluid, ~~and~~ spatially isotropic, and homogeneous. The coefficient a^2 of the spatial part of the stress-energy tensor signifies the spatial expansion of the universe.

The FEs describe the expansion of the universe in terms of the scale factor and its derivatives, given its mass-energy content and curvature. The first FE is given by

$$H^2 = \left(\frac{\dot{a}}{a}\right)^2 = \frac{8\pi G\rho}{3c^2} - \frac{kc^2}{a^2} + \frac{\Lambda c^2}{3}, \tag{3.2a}$$

where H = $\dot{a}/a$ is the Hubble parameter~~, $\dot{} = \partial/\partial t$ denotes a derivative with respect to cosmic time t,~~ and k is the curvature parameter of the universe, which can be positively curved (+1), flat (0), or negatively curved (−1). The above equation compares the rate of the universe's expansion to its scale and relates it to the mass-energy content of the universe. The second FE is given by

$$\frac{\ddot{a}}{a} = -\frac{4\pi G}{3c^2}\left(\rho + 3p\ \cancel{\frac{3p}{c^2}}\right) + \frac{\Lambda c^2}{3}, \tag{3.2b}$$

which compares the acceleration of the universe's expansion to its scale. ~~The Friedmann–Lemaître–Robertson–Walker (FLRW) metric — the metric for a spatially isotropic, homogeneous, and flat universe — is derived from the EFEs and FEs. The corresponding line element ds is given by~~

~~$$ds^2 = -c^2dt^2 + a(t)^2d\mathbf{x}^2, \tag{3.3}$$~~

~~where t is cosmic time and $\mathbf{x}$ is a set of 3D spatial coordinates. Here, we use the (− + + +) metric signature. If instead we were to use the (+ − − −) metric signature, the signs of the temporal and spatial terms in Equation 3.3 would switch. For more on the metric tensor, see the Appendix. And by using the FLRW metric, the stress-energy tensor components become $T_{00} = \rho$ and $T_{ii} = a^2p$.~~ The FEs are important because different dark energy models use them to predict different evolutions of the universe (i.e., different Hubble parameters). ~~Another important equation for understanding the dynamics of dark energy is its~~The EoS w of dark energy is~~,~~ given by the ratio of pressure p to energy

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density ρ for a perfect fluid: $w = p/\rho$. In ΛCDM, the EoS for dark energy equals −1, meaning ~~that the~~ its pressure and energy density ~~of dark energy~~ are constant in time and $p = -\rho$. Other components of the universe — including radiation, ordinary matter, and dark matter — have different values of w, so w itself is not unique to dark energy. The EoS is useful because it determines the dynamics of dark energy and how it drives the expansion of the universe. It is also one of the many important parameters measured by probes (T. M. C. Abbott et al., 2025; Abdul-Karim, Aguilar, et al., 2025).

While the EFEs and FEs are well-established equations in that the EFEs give extremely accurate predictions of the behavior of astronomical objects and that the FEs are derived from the EFEs, ~~(assuming an isotropic and homogeneous universe),~~ huge issues exist in experimentally verifying the value of H, and disagreements arise in different physical theories of Λ.

3.1 Hubble tension

In ΛCDM, dark energy is modeled using a cosmological constant Λ. Astronomers use various methods to determine the present-day value of $H(t~~a~~)$, called the Hubble constant H_0, with methods including measuring distances using standard candles such as Cepheids (pulsating stars) and SNe Ia, as well as measuring redshifts of galaxies (Riess et al., 1998, 2019). These methods then help determine the scale factor a and solve for H via the FEs. ~~However, a major disagreement exists between measurements of the value of H_0. The 2018 *Planck* collaboration determined a value of H_0 = 67.36 ± 0.54 km.s^{-1}.Mpc^{-1}, whereas the 2024 SH0ES collaboration measured a value of 73.17 ± 0.86 km.s^{-1}.Mpc^{-1} at a >5σ tension with the *Planck* collaboration (Zhou et al., 2025). The cause of this discrepancy is unknown and is a huge problem for cosmology as it indicates potential systematic errors in our measurements or a fundamental flaw in ΛCDM, requiring new developments in measurement techniques and mathematical models. Different measurements of H_0 from 2001 to 2022 are shown with error bars in Figure 1.~~

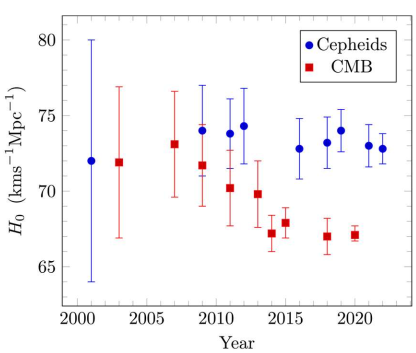

Figure 1. Various measurements of the Hubble constant H_0 from Cepheids (blue) and CMB data (red) from 2001 to 2022. A major discrepancy between the measured value of H_0 exists. Reproduced from (Perivolaropoulos & Skara, 2022).

However, a major disagreement exists between measurements of the value of H_0. The 2018 *Planck* collaboration determined a value of H_0 = 67.36 ± 0.54 km s^{-1} Mpc^{-1}, whereas the 2024 SH0ES collaboration measured a value of 73.17 ± 0.86 km s^{-1} Mpc^{-1} at a $>5\sigma$ tension with the *Planck* collaboration (Zhou et al., 2025). The cause of this discrepancy is unknown and is a huge problem for cosmology as it indicates potential systematic errors in our measurements or a fundamental flaw in ΛCDM, requiring new developments in measurement techniques and mathematical models. Different measurements of H_0 from 2001 to 2022 are shown with error bars in Figure 1.

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3.2 Cosmological constant problem

The cosmological constant problem refers to a discrepancy between the observed and predicted values of the vacuum energy density (vacuum energy per cubic meter) $\rho_\Lambda = \Lambda c^4/8\pi G$ (and hence Λ). Quantum field theory (QFT) ~~is a theory comprising~~comprises ~~principles from~~ special relativity, quantum mechanics, and classical field theory; it is currently one of the most complete and successful physical theories and is used to formulate the Standard Model of particle physics. ~~Weinberg (1989) calculated that the predicted value of Λ in QFT was around 120 orders of magnitude larger than the observed value. This implied that there was a flaw in QFT or errors in observational methods, indicating a need for further theoretical and empirical development in physics.~~ In QFT, the zero-point energy of a vacuum (also called the vacuum energy) refers to the minimum amount of energy a vacuum can have. Dark energy is a form of vacuum energy where the vacuum energy density is constant, which means that the total amount of dark energy increases as the universe expands. Weinberg (1989) calculated that the predicted value of Λ in QFT (2×10^{109} J m^{-3}) is around 120 orders of magnitude larger than the observed value (~~The observed value of ρ_Λ is~~ 2×10^{-11} J m^{-3})~~, but the predicted value from QFT is 2×10^{109} J m^{-3}, corresponding to a discrepancy of 120 orders of magnitude~~ (Yoo & Watanabe, 2012). This discrepancy is a massive problem as it demonstrates the current incompatibility between theory and cosmological observations; this may require significant reformulation of QFT or theories of cosmology, fundamentally changing our understanding of dark energy.

3.3 Coincidence problem

Current measurements suggest that the universe is extremely flat (i.e., the curvature parameter k is essentially zero). The curvature of the universe is affected by its mass-energy content. The critical density ρ_c is defined as the energy density (of all the content of the universe, including matter, radiation, and dark energy) required for $k = 0$, where we define the ratio ρ/ρ_c as the density parameter Ω. The Wilkinson Microwave Anisotropy Probe (WMAP) and the Supernova Legacy Survey (SNLS) measured $\Omega = 1$ with around 1% deviation (Spergel et al., 2007). Observations also suggest that the density parameter for matter is $\Omega_m \approx 0.3$ and that for dark energy is $\Omega_\Lambda \approx 0.7$. The coincidence problem then asks the question of why Ω_m and Ω_Λ are of the same order of magnitude today despite evolving very differently in the past. Since the universe is measured to be very flat (i.e., $\Omega \approx 1$ and $\rho \approx \rho_c$), this suggests that if ρ_m or ρ_Λ were to slightly deviate away from the critical value, over time, this would result in drastically different evolutions of the universe, including the Big Crunch (where gravity collapses the universe) and the Big Freeze (where the universe grows cold) (Zlatev et al., 1999). Problems such as the Hubble tension, the cosmological constant problem, and the coincidence problem imply the need for new models of dark energy, with one such being quintessence.

4 Quintessence

There are two general types of modified dark energy models. Those that change the left side of the EFEs (Equation 3.1) result in modified gravity models, and those that change the right side result in modified matter models (Tsujikawa, 2013; Yoo & Watanabe, 2012). This paper will focus on quintessence (a term coined by Caldwell et al. (1998)), which is a modified matter model. It is one of the simplest alternatives to Λ involving a change to the stress-energy tensor $T_{\mu\nu}$. Quintessence is postulated as a “fifth fundamental force” that ~~couples~~ interacts gravitationally ~~to~~ with matter and dismisses the requirement of Λ, instead modelling dark energy using a ~~dynamic (~~time-dependent~~)~~, ~~slowly evolving,~~ and ~~possibly~~ (on large scales) homogeneous ~~inhomogeneous~~ scalar field $\varphi(x^\mu)$ ~~(though assumed to be homogeneous in this paper, which is an accurate approximation over large scales), that couples only gravitationally to matter,~~ with spacetime coordinates x^μ and a potential energy V(φ) that gradually decreases to a minimum (Steinhardt, 2003; Yoo & Watanabe, 2012). This contrasts with ΛCDM in that its~~, where dark energy~~ dark energy EoS remains constant through time. Due to Λ's constancy, ΛCDM is easier to falsify since any significant deviation from $w = -1$ would strongly indicate its implausibility, whereas quintessence is dynamic and can take on multiple evolutions of w, so it is harder to falsify.

One of the weaknesses of q~~Q~~uintessence is that it does not solve the Hubble tension nor the cosmological constant problem, but it does partially solve~~s~~ the coincidence problem, which arises due to the energy density of dark energy ρ_Λ and matter ρ_m in the universe decreasing at different rates, hence requiring their initial values to be fine-tuned (Zlatev et al., 1999). Quintessence addresses this problem by providing a “tracking” behavior of φ where the quintessence EoS roughly equals the EoS for radiation during the radiation-dominated epoch and that for matter during the matter-dominated epoch of the universe, with the EoS of quintessence only recently becoming dominant. This allows a wide range of initial values for φ to lead to the same evolution of w. However, this means that the potential V must still ~~in turn~~ be fine-tuned~~, which shifts the coincidence problem to “why this potential?”~~. Despite its weakness, physicists still consider quintessence as a possible dark energy candidate because if the equation of state of dark energy turns out to be non-constant, then quintessence and ΛCDM can be observationally distinguished. We can also study the physical motivations behind quintessence and apply some of its principles to other dark energy models. Quintessence may also reconcile problems that arise in particle physics and effective field theory whilst also being more physically motivated than ΛCDM (Souza et al., 2025); this is because Λ is a term arbitrarily added to accommodate for the universe's accelerated expansion, whereas the universe's acceleration and the dark energy EoS may vary with cosmic time, which a dynamical scalar field may more accurately describe.

Dark energy possesses an energy density and pressure, and we require that a mathematical description of these quantities remain the same regardless of what coordinates are used. To encapsulate this, one uses t~~T~~he stress-energy tensor $T_{\mu\nu}$, which, for the quintessence field φ~~,~~, is given by (Copeland et al., 2006; Li et al., 2011)

$$T_{\mu\nu} = \partial_\mu\varphi\,\partial_\nu\varphi - g_{\mu\nu}\left[\frac{1}{2}\partial^\alpha\,\partial_\alpha\varphi + V(\varphi)\right], \tag{4.1}$$

Formatted: Font: Cambria, Font color: Text 1, (Asian) Chinese (Simplified, Mainland China) where $g_{\mu\nu}$ is the metric tensor, and ∂_μ ~~$= \partial/\partial x^\mu$~~ and ∂^μ ~~$= \partial/\partial x_\mu$~~ denote ~~the covariant and contravariant~~ 4-gradients (which are four-dimensional derivatives in time and space), respectively. The above equation shows how the pressure and energy of the quintessence field depends on its rate of change in spacetime and its potential energy. The stress-energy tensor can be used to determine the EoS of quintessence, which can then predict the universe's expansion history and fate. ~~We~~ It is assumed that pressure and energy density are measured in the local rest frame of an observer comoving with the quintessence field; that is, the observer "moves along" with the expansion of the universe such that it does not appear to be expanding.~~;~~ To achieve this, the factor of~~otherwise, an additional factor of a^2 is included. To get rid of~~ a^2 is removed ~~, we raise one index of $T_{\mu\nu}$ with the inverse FLRW metric $g^{\mu\nu}$ = diag($-1,a^{-2},a^{-2},a^{-2}$)~~ such that $T_{\mu\nu} = \mathrm{diag}(\rho, a^2p_x, a^2p_y, a^2p_z)$ becomes $T^\mu{}_\nu = \mathrm{diag}(-\rho, p_x, p_y, p_z)$. By assuming a spatially homogeneous field φ (i.e. its spatial derivatives are zero), we see from Equation 4.1~~2~~ that the energy density $\rho = -T^0{}_0$ and pressure $p = T^i{}_i$ for i = 1, 2, 3 of the field φ in the local rest frame of an observer are given by

~~$\rho = T_{00} = \frac{1}{2}\dot{\varphi}^2 + V(\varphi), \qquad p = T_{ii} = \frac{1}{2}\dot{\varphi}^2 - V(\varphi),$~~

$$\rho = -T^0{}_0 = -\partial^0\varphi\,\partial_0\varphi + g^0{}_0\left[\frac{1}{2}\partial^\alpha\,\partial_\alpha\varphi + V(\varphi)\right] = \dot{\varphi}^2 - \frac{1}{2}\dot{\varphi}^2 + V(\varphi),$$
$$p = T^i{}_i = \partial_i\varphi\,\partial_i\varphi - g^i{}_i\left[\frac{1}{2}\partial^\alpha\,\partial_\alpha\varphi + V(\varphi)\right] = 0 + \frac{1}{2}\dot{\varphi}^2 - V(\varphi), \qquad (4.2)$$

leading to the following EoS:

$$w = \frac{p}{\rho} = \frac{\dot{\varphi}^2/2 - V(\varphi)}{\dot{\varphi}^2/2 + V(\varphi)}. \qquad (4.3)$$

We see from Equation 4.3 that w can only range from -1 (when V tends to infinity, making $\dot{\varphi}^2/2$ negligible) ~~-~~to 1 (when $\dot{\varphi}^2/2$ tends to infinity, making V negligible). The EoS can also be written using the Chevallier-Polarski-Linear (CPL) parametrization (Chevallier & Polarski, 2001), a common parametrization found in literature which uses the cosmological scale factor a, w_0 (the present-day value of w with a = 1) and $w_a = -dw/da$ evaluated at a = 1 ~~(corresponding to the negative rate of change of w)~~; it is a first-order Taylor expansion around a = 1 and linearly approximates non-linear solutions to w for high-a (i.e. the plot of w against a using CPL is a straight line). The CPL parametrization is given by (Bayat & Hertzberg, 2025)

$$w(a) = w_0 + w_a(1 - a). \qquad (4.4)$$

From Equation 4.4, we see that w_0 = 0 and w_a = 1 for ΛCDM. We refer to the cosmological model that assumes cold dark matter (CDM) and the CPL parametrization as the w_0w_aCDM model. For quintessence, since $w \geq -1$, any region ~~of~~ where $w < -1$ is called the phantom region (which corresponds to the phantom dark energy model). Thus, the line $w = -1$ is called the phantom divide line (PDL). To compare different dark energy models, ~~we~~ this paper looks at the predicted evolution of w with respect to a and z and how it correlates with observational data. There are different classes of quintessence models, with the two main types being freezing (w_a < 0) and thawing (w_a > 0) models. The idea behind having differing models of quintessence is that, even though the potential field decreases towards a minimum, its rate of decrease could vary at different epochs of the universe, and this results in different behaviors of the quintessence EoS and predicted evolutions of the universe. The scalar field φ follows the same equation of motion in both types (Bayat & Hertzberg, 2025; Chevallier & Polarski, 2001; Perivolaropoulos & Skara, 2022; Spergel et al., 2007):

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$$\ddot{\varphi} + 3H\dot{\varphi} + V'(\varphi) = 0, \tag{4.5}$$

where H is Hubble's parameter and $V' = d/d\varphi$ denotes ais the derivative of V with respect to φ. The above equation determines how the scalar field φ will evolve, given its acceleration $\ddot{\varphi}$, its effect due to Hubble friction $3H\dot{\varphi}$, and the rate of change of its potential energy V (which is what decelerates φ). The above equation can then be used to deduce the predicted evolution of w for quintessence models The term $3H\dot{\varphi}$ is called Hubble friction since it acts as a dampening term and is what decelerates φ; V' causes the acceleration of φ. An example freezing model potential is given by

$$V_f V(\varphi) = M^{4+\beta\alpha}\varphi^{-\beta\alpha}, \tag{4.6}$$

where M is the mass parameter (chosen a quantity that so thatmakes the quintessence scalar field energy density matches match the observed value of the dark energy density ρ_Λ) constrained to be less than or approximately 1.78×10^{-39} kg (Khoury, 2013; Khoury & Weltman, 2004) and $\beta\alpha > 0$ is a constant. The idea behind freezing models is that in the early stages of the universe, $w > -1$ and V_f had already been decreasing towards a minimum, but as time passes, its rate of decrease slows down and tends towards $w = -1$. This is shown through the exponential term $\varphi^{-\beta}$. An example thawing model potential is given by

$$V_t V(\varphi) = M^4\left[1 + \cos\left(\frac{\varphi}{f_\alpha f}\right)\right], \tag{4.7}$$

where f_α f is a constant representing the width of V_t. The idea behind freezing models is that in the early stages of the universe, $w > -1$ and V had already been decreasing towards a minimum, but as time passes, its rate of decrease slows down and tends towards $w = -1$. The idea behind tThawing models, on the other hand, haveis that, in a sense "opposite" to freezing models, $w \approx -1$ and V_t remains constant in the early stages, and and then w increases with time as V_t starts to decrease to its minimum (Caldwell & Linder, 2005)(Caldwell & Linder, 2005; Tsujikawa, 2013; Yoo & Watanabe, 2012).(Tsujikawa, 2013; Yoo & Watanabe, 2012) This occurs due to the Hubble friction term in Equation (4.5), $3H\dot{\varphi}$, which causes the initial dampening of V_t (causing its initial constancy) (Caldwell & Linder, 2005; Tsujikawa, 2013; Yoo & Watanabe, 2012). The EoS of the freezing potential (Equation 4.6) is given by (Chiba et al., 2013; Tsujikawa, 2013)

$$w_f w(a) = w_0 + \sum_{n=1}^{\infty} \frac{(-1)^{n-1} w_0 (1 - w_0^2)}{1 - w_0(n+1) + 2nw_0^2(n+1)} \left[\frac{\Omega_\varphi \Omega(a)}{1 - \Omega_\varphi \Omega(a)}\right]^n, \tag{4.8}$$

where $\Omega_\varphi(a) = \rho_\varphi \rho/(3H^2M_{pl}^2)$ is the field density parameter of φ, ρ_φ is the energy density of φ, H is the Hubble parameter, and M_{pl} is the Planck mass. The term $\Omega_\varphi(a)/[1 - \Omega_\varphi(a)]$ "tracks" how much dark energy and matter dominates throughout the universe's history and thus determines the rate of evolution of w_f. The 3rd-order analytical solution to $w_f w(a)$ for the freezing potential (Equation 4.8) is plotted in Figure 2.

The EoS of the thawing potential (Equation 4.7) is given by

$$w_t(a) = -1 + (1 + w_0)a^{3(K-1)}\tilde{F}(a), \tag{4.9}$$

where we define the quantities K and F(a) as

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$$K \equiv \sqrt{1 - \frac{4M_{\rm pl}^2 V''}{3V}},$$
$$F(a) \equiv \sqrt{1 + (\Omega_{\varphi 0}^{-1} - 1)a^{-3}}, \qquad (4.10)$$

such that $\tilde{F}(a)$ is given more concisely by

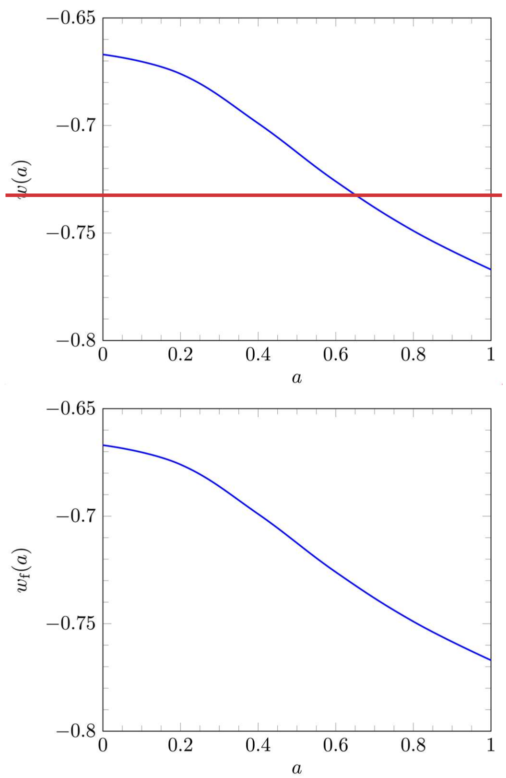

Figure 2. Plot of the 3rd-order analytical solution to $w_f(a)$ (Equation 4.8) against the scale factor a for the freezing potential (Equation 4.6) with ~~α~~ β = 1. Reproduced from (Tsujikawa, 2013).

$$\tilde{F}(a) = \left[\frac{(K-F)(F+1)^K + (K+F)(F-1)^K}{\left(K - \Omega_{\varphi 0}^{-1/2}\right)\left(\Omega_{\varphi 0}^{-1/2}+1\right)^K + \left(K + \Omega_{\varphi 0}^{-1/2}\right)\left(\Omega_{\varphi 0}^{-1/2}-1\right)^K}\right]^2, \qquad (4.11)$$

~~The EoS of the thawing potential (Equation 4.7) is given by~~

~~$$w(a) = -1 + (1 + w_0)a^{3(K-1)}F(a), \qquad (4.9)$$~~

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$$K \equiv \sqrt{1 - \frac{4M_{\rm pl}^2 V''}{3V}},$$

$$\tilde{F}(a) \equiv \sqrt{1 + (\Omega_0^{-1} - 1)a^{-3}}, \quad (4.10)$$

such that $F(a)$ is given by

$$F(a) =, \quad (4.11)$$

where $\Omega_{\varphi 0} \approx 0.7$ is the present-day value of Ω_φ for dark energy (Tsujikawa, 2013). The term $a^{3(K-1)}$ controls how fast w_t grows; a larger K means a faster thawing. K describes the curvature of V (given by the second derivative of V) and thus also correlates with the rate at which w_t grows. F depends on the curvature parameter of dark energy and tends towards $1/\Omega_{\varphi 0}$ as a goes to 1. $\tilde{F}$ therefore takes into account both the shape of the potential energy of quintessence and its relative dominance compared to matter. The above quantities are arbitrary parameters simply defined to compactify Equation 4.9. Analytical solutions to w_t $w(a)$ for the thawing potential (Equation 4.9) with K = 1.9, 2.9, and 8.2 are plotted in Figure 3.

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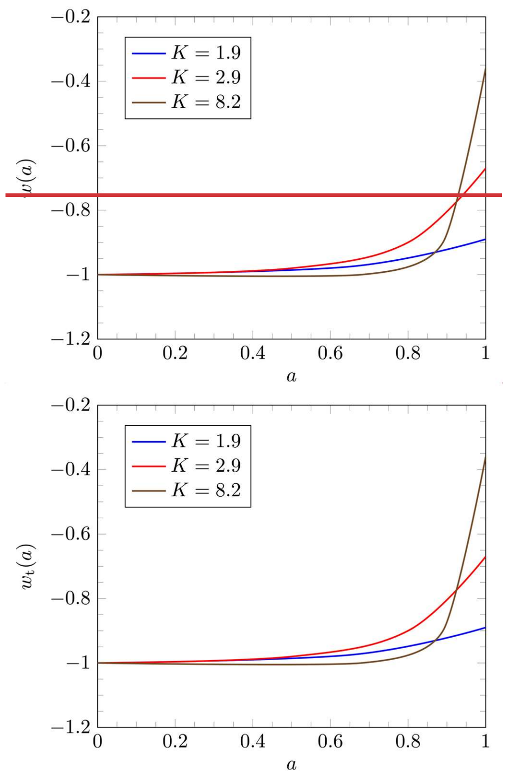

Figure 3. Plot of the analytical solution to w_t~~w~~(a) (Equation 4.9) against the scale factor a for the thawing potential (Equation 4.7) with K = 1.9, 2.9, and 8.2. Reproduced from (Tsujikawa, 2013).

5 Results and discussion

~~In t~~The previous ~~sub~~section~~, we discussed~~ reviewed ~~the different classes of the~~freezing and thawing models of quintessence ~~model of dark energy — namely, the freezing and thawing models~~. In 2025, the Dark Energy Spectroscopic Instrument (DESI) released data from BAO measurements from more than 14 million galaxies and quasars, which were used to constrain various cosmological parameters~~, including w, w_0, and w_a~~ (Abdul-Karim, Aguilar, et al., 2025). ~~We compare t~~The EoS of freezing and thawing models with CPL constraints from the DESI DR2 data are compared; these are plotted in Figure 4.

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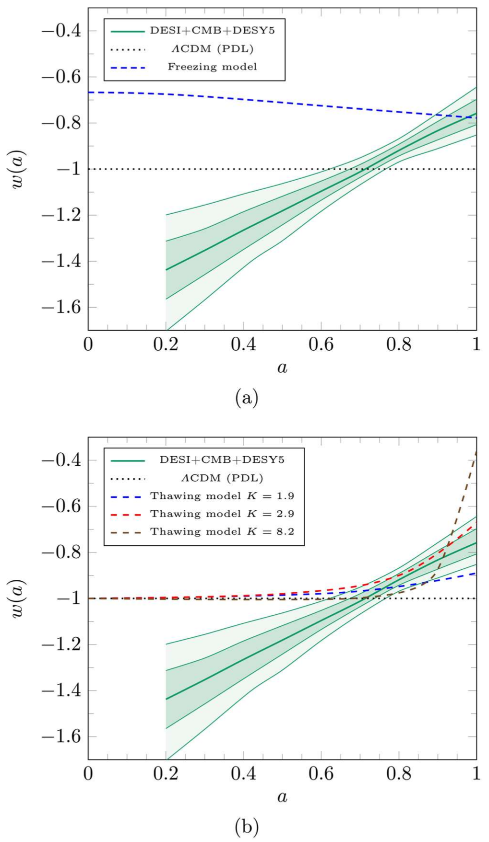

Figure 4. Plots of the EoS $w(a)$ using CPL (green) against scale factor a for freezing (a) and thawing (b) quintessence models for K = 1.9 (blue), 2.9 (red), and 8.2 (brown) with observational constraints from the 2025 DESI DR2. The solid green curve represents the observed best fit of w for CPL, and the dark- and light-green bands around it represent a 68% and 95% confidence region for CPL, respectively. Reproduced from (Abdul-Karim, Aguilar, et al., 2025).

From Figure 4, we see that the ΛCDM model falls within the green bands only within around $0.6 < a < 0.75$, but does not deviate as significantly when compared to the freezing model. Thus, ΛCDM is not completely ruled out based solely on CPL constraints. We see from Figure 4a that the freezing model deviates significantly for $a < 0.8$ — well beyond the 95% confidence region when compared to DESI measurements, thus not favored by observations. The thawing model in Figure 4b with K = 2.9 falls within the 68% and 95% confidence regions for $0.7 < a < 1$, but deviates more significantly for $a < 0.5$. However, deviations are expected since CPL is only a linear parametrization in a, and many quintessence potentials lead to a non-linear w (with examples including the freezing and thawing models in this paper; see Figures 2 and 3). This means that CPL only approximates a local region of a predicted EoS (Linden & Virey, 2008), as seen in Figure 4 with good approximation around $0.6 < w < 1$.

From Figure 4, we also see that at around $a \approx 0.7$, it is likely that w crossed the PDL, corresponding to a transition between quintessence and phantom dark energy models. Phantom dark energy is modelled with a potential ϕ and a negative kinetic energy component; it is characterized by $w \leq -1$. Its EoS is given by

$$w = \frac{p}{\rho} = \frac{-\dot{\phi}/2 - V(\phi)}{-\dot{\phi}/2 + V(\phi)}. \tag{5.1}$$

However, even a purely phantom dark energy model would not satisfy the green bands in Figure 4 for $a > 0.7$. An analysis conducted by Lodha et al. (2025) also showed that freezing and thawing models lack strong support from observational data. This indicates a main weakness of quintessence, which is that its predictions of the equation of state does not correlate very strongly with observational evidence and deviates for smaller values of a. Feng et al. (2005) proposed a quintom model (a portmanteau of quintessence and phantom) where w was allowed to transition between quintessence and phantom behavior.

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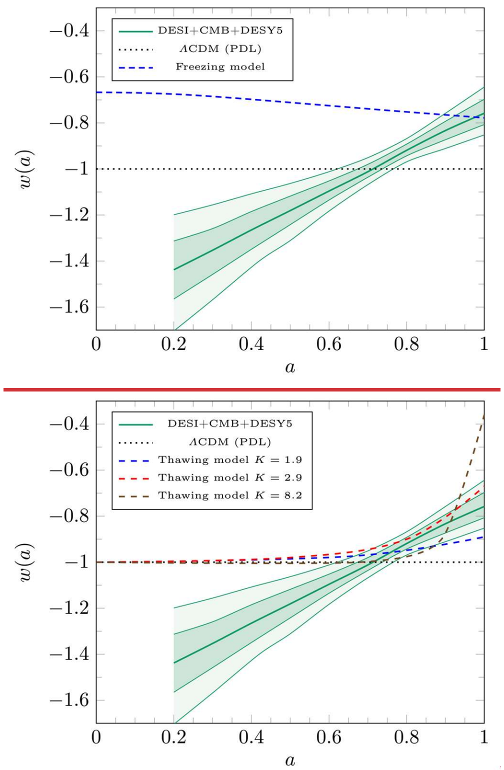

Figure 4. Plots of the EoS $w(a)$ using CPL (green) against scale factor a for freezing (top) and thawing (bottom) quintessence models for K = 1.9 (blue), 2.9 (red), and 8.2 (brown) with observational constraints from the 2025 DESI DR2. The dark- and light-green bands around the solid green curve represent a 68% and 95% confidence region for CPL, respectively. The solid green curve represents the observed best fit of w for CPL. Reproduced from (Abdul-Karim, Aguilar, et al., 2025).

From Figure 4, we see that the ΛCDM model falls within the green bands only within around $0.6 < a < 0.75$, but does not deviate as significantly when compared to the freezing model and is thus not completely ruled out based solely on CPL constraints. We see that the freezing model deviates significantly for $a < 0.8$ — well beyond the 95% confidence region when compared to DESI measurements, thus not favored by observations. The thawing model with $K = 2.9$ falls within the 68% and 95% confidence regions for $0.7 < a < 1$, but deviates significantly starting around $a = 0.5$. However, deviations are expected since CPL is only a linear parametrization (in a) and many quintessence potentials lead to a non-linear w; that is, CPL only approximates a local region of a predicted EoS (Linden & Virey, 2008), as seen in Figure 4 with good approximation around $0.6 < w < 1$.

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modelled with a potential ϕ and a negative kinetic energy component; it is characterized by $w \le -1$. Its EoS is given by

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However, even a purely phantom dark energy model would not satisfy the green bands in Figure 4 for $a > 0.7$. An analysis conducted by Lodha et al. (2025) also showed that freezing and thawing models lack strong support from observational data, indicating that pure quintessence behavior is relatively unfavored by observations. Feng et al. (2005) proposed a quintom model (a portmanteau of quintessence and phantom) where w was allowed to transition between quintessence and phantom behavior.

Higher-order parametrizations based on CPL have also been proposed and analyzed to better approximate dark energy models (Nesseris et al., 2025). An extended parametrization based on CPL is given by

$$w(a) = w_0 + w_a(1-a) + w_b(1-a)^2, \qquad (5.2)$$

where w_0 is the present-day value of w, $w_a = -dw/da$ at $a = 1$, and $w_b = -(d^2w/da^2)/2$ at $a = 1$. Models assuming CDM and the extended CPL parametrization given by Equation 5.2 are dubbed $w_0w_aw_b$CDM. Figure 5 compares freezing and thawing quintessence EoSs with $w_0w_aw_b$CDM constraints from the 2024 DESI DR1.

The DESI collaboration also provided measurements of the possible ranges for w_0 and w_a for the CPL parametrization of w (Abdul-Karim, Aguilar, et al., 2025), shown in Figure 6. We see that the point with $w_0 = -1$ and $w_a = 0$, corresponding to ΛCDM, falls outside the 68% and 95% confidence regions, with the DESI data rejecting ΛCDM with confidence 2.8–4.2σ. We also see that the DESI data provides relatively consistent results favoring $w_0 > 1$ and $w_a < 0$, further indicating deviation from ΛCDM. Wolf et al. (2024) showed that there is minimal overlap between the predicted and observed regions for w_0 and w_a for thawing models. However, their analysis used the potential $V(\varphi) = V_0 + m^2\varphi^2/2$, which they argued represents various potentials and covers a wide range of values for w_0 and w_a.

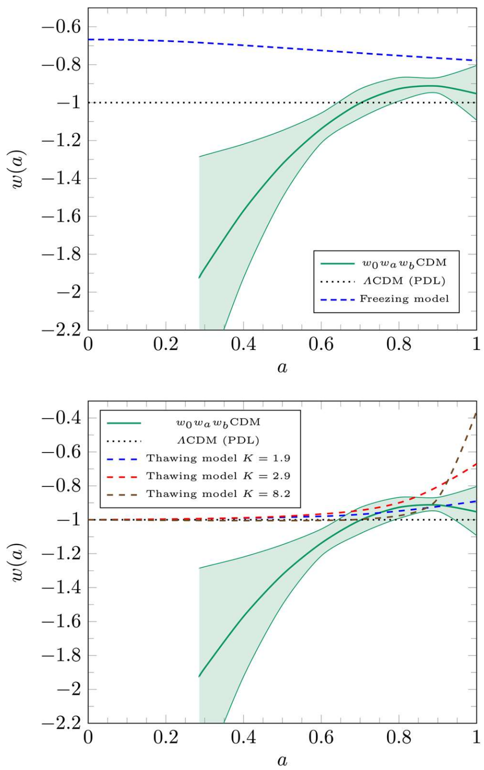

Figure 5. Plots of the EoS $w(a)$ using extended CPL parametrization against scale factor a for freezing (top) and thawing (bottom) quintessence models with observational constraints from the 2024 DESI DR1. The solid green curve represents the observed best fit of w under extended CPL parametrization , and tThe green band around the solid green curveit represents a 68% confidence region for extended CPL. The solid green curve represents the observed best fit of w under extended CPL parametrization. Reproduced from (Abdul-Karim, Aguilar, et al., 2025) and (Nesseris et al., 2025).

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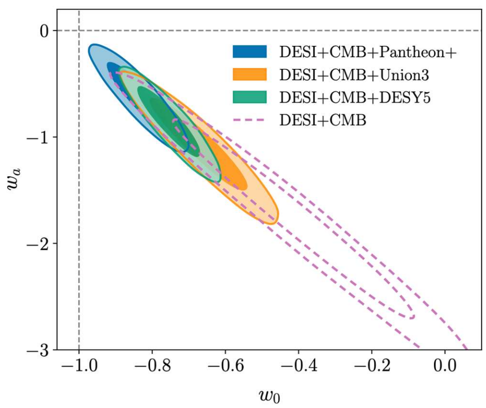

Figure 6. Contour plots of the posterior distributions of w_0 and w_a, with the darker and lighter regions (or the smaller and larger curves) representing a 68% and 95% confidence level, respectively. The intersection of the grey dashed lines represents the values of w_0 and w_a for ΛCDM. Image from (Abdul-Karim, Aguilar, et al., 2025).

Lodha et al. (2025) conducted further analysis of the DESI DR2 data to test the consistency of dynamic dark energy models ~~without~~ for a broad range of~~a~~ functional forms of w. They compared various parametrizations, including the CPL, Barboza-Alcaniz (BA), exponential (EXP), logarithmic (LOG), and Jassal-Bagla-Padmanabhan (JBP) parametrizations, also including an EoS written in terms of Chebyshev polynomials. They found that the resultant form of w is still relatively consistent with observations, having little dependence on ~~the fitting of data and plausibility of dynamical dark energy models had little dependence on parametrization~~which parametrization is used for a dynamical dark energy model. ~~, which is strong evidence for the implausibility of ΛCDM for dark energy.~~ Lodha et al. also used a binning approach, an unbiased method of comparing w at different redshift intervals without assuming a functional form of w. This ensures that ~~we can reconstruct~~ the evolution of the universe's expansion (e.g., through w and the FEs) can be reconstructed with minimal presumptions about the specific model of dark energy. Figures 7 and 8 compare the data from binning and constraints for w_0w_aCDM and $w_0w_aw_b$CDM, respectively.

~~-~~These results suggest evidence for a quintom scenario (i.e., w_0w_aCDM, $w_0w_aw_b$CDM, and the like), with $w_0w_aw_b$CDM appearing to fit the binning data slightly better than w_0w_aCDM (although with more uncertainty at low-a regions). ~~However, we must keep in mind that~~Despite this, alternative parametrizations or models are still not ruled out — including ΛCDM — since current observations are unable to measure w precisely for non-local (smaller) values of a. CPL and extended CPL are also only

Taylor approximations about $a = 1$, so higher-order behavior in a of w potentially cannot be captured by these parametrizations.

In the analysis of the DESI data by Gialamas et al. (2025), however, it was argued that although phantom behavior is suggested by observational data, it is not statistically robust as pure quintessence behavior with $w \geq -1$ is still supported with 2σ confidence. In their analysis, they included a Higgs-like quintessence (HLQ) model without phantom behavior; despite CPL matching observational data slightly better than HLQ, HLQ is motivated through physical theory, unlike CPL, which was developed only to parametrize and fit observational data (Gialamas et al., 2025).

The reason for varied predicted evolutions of w from different dark energy parametrizations and models is due to degeneracy, which is the idea that many different models of dark energy can give rise to the same cosmological observations, and our current observational data is not sensitive enough to distinguish the different scenarios for w at high-z (low-a) (Marttens et al., 2020). ~~Nevertheless, we conclude with fairly high confidence that the current data favors dynamical dark energy over ΛCDM.~~

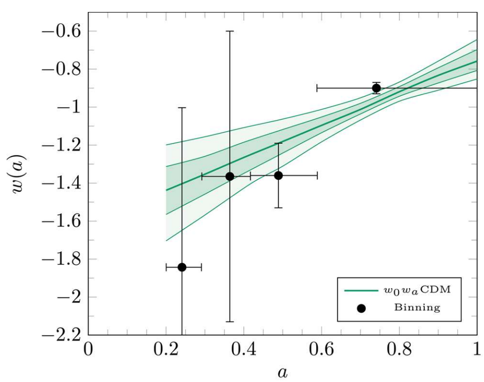

Figure 7. A comparison between w_0w_aCDM constraints (with the dark- and light-green contours representing 68% and 95% confidence regions) and data from the binning method. The black bins represent a 1σ error. Reproduced from (Abdul-Karim, Aguilar, et al., 2025).

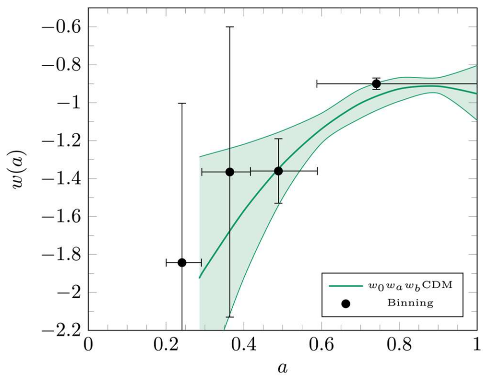

Figure 8. A comparison between $w_0w_aw_b$CDM constraints (with the dark-green contours representing a 68% confidence region) and data from the binning method. The black bins represent a 1σ error. Reproduced from (Abdul-Karim, Aguilar, et al., 2025) and (Nesseris et al., 2025).

From all the above figures and available data, this paper concludes that the current data favors dynamical dark energy over ΛCDM with fairly high confidence, although current instruments are not sensitive enough to differentiate between particular dark energy models, so multiple dark energy models leading to similar EoS or predicted evolutions of the universe cannot be ruled out. The EoS from ΛCDM also does not deviate significantly enough from observational data for this paper to conclude with high confidence that ΛCDM is implausible. Moreover, using data from the unbiased binning method for measuring the dark energy EoS, this paper concludes that there is some indication of the EoS involving phantom behavior (i.e. that it crosses the PDL).

Even if we do manage to further constrain the EoS, issues like the Hubble tension would still prevent a conclusive decision on a correct dark energy model. The conflict between QFT, general relativity, and observational measurements of the vacuum energy density further implies a fundamental flaw in our mathematical descriptions of dark energy. Quintessence also does not solve the coincidence problem fully as it still requires fine tuning, essentially moving the goalpost from "why this specific value?" to "why this potential?", and thus more physically motivated models like chameleon particles or modified gravity models are more likely alternatives. A development in the theory behind dark energy that explains the initial potential value would strengthen the plausibility of quintessence.

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6 Conclusion

In this paper, ~~we discussed~~ the main pieces of evidence supporting the accelerated expansion of the universe were discussed and ~~motivated~~ the development of alternative dark energy theories is motivated by discussing current issues with ΛCDM. ~~We then reviewed the~~Then, the core concepts of ΛCDM~~, including the EFEs, FEs, and the Λ model for dark energy,~~were reviewed and ~~explored~~ the basic physics of quintessence was explored. ~~We compared t~~The predicted EoS from quintessence to observational data from the DESI collaboration was compared and it was found that quintessence is a more preferred model than ΛCDM. However, due to strong evidence for a phantom crossing, ~~we~~ it is concluded that quintessence nor phantom fully match observational data. ~~We instead find~~It is instead found that models including w_0w_aCDM and $w_0w_aw_b$CDM fit observational data better, with dynamical dark energy models being more favorable than constant ones.

~~However~~However, this paper is not a comprehensive review of all dark energy models; ~~we only focused on~~only particular modified matter models (with an emphasis on quintessence) and how closely their predictions fit observational data were focused on. Hence, further comparisons and research into other modified matter models (such as ~~phantom,~~ K-essence, Chaplygin gas, and chameleon) and modified gravity models (such as $f(R)$ gravity, modified Newtonian dynamics, and DGP braneworld) (Yoo & Watanabe, 2012) are needed. Furthermore, because current observations do not allow high distinguishability between certain dark energy models and parametrization, this paper concludes that ΛCDM is still a feasible model, but is seemingly slowly being ruled out. Further development in observational techniques are required.

This paper also mainly focused on comparing predicted evolutions of the EoS for each model; there are, however, other methods of assessing the plausibility of a model, such as its mathematical complexity and the number of parameters required to describe the model. ~~We defer f~~Further analysis of dark energy models, especially of phantom and quintom, is deferred to future work.

Acknowledgements

I am sincerely grateful to my mentor, Dr. Chima McGruder, and teaching assistant, Catherine Peretti, for delivering sessions on basic astrophysics knowledge, guiding me through the research process, and reviewing drafts of my paper. I am also grateful to my peers who took the IRIS Indigo Program for providing valuable feedback on my paper.

This research paper contains data from the DESI collaboration. Any opinions and conclusions written in this paper belong to the author and do not necessarily reflect the views of the U.S. National Science Foundation, the U.S. Department of Energy, or any other related funding agencies.

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Appendix

~~Appendix~~

This paper uses special notations for derivatives. An overhead dot $\dot{}$ = $\partial/\partial t$ denotes a derivative with respect to cosmic time t and an apostrophe ' = $d/d\varphi$ denotes a derivative with respect to the scalar field φ.

Einstein's summation notation (ESN) is a convenient way of denoting sums of indexed terms. Consider a vector $\boldsymbol{v}$, which can be expressed as a linear combination of n basis vectors $\boldsymbol{e}_i$ with n corresponding vector components v^i (where i is the index denoting the i-th object). Upper and lower indices denote contravariance and covariance with respect to basis vectors, respectively, (so a covector $\boldsymbol{\alpha} = \alpha_i \boldsymbol{\varepsilon}^i$ has covariant components α_i and contravariant basis covectors $\boldsymbol{\varepsilon}^i$). ESN simplifies the notation in such a way that the usual "capital sigma" symbol in front of a sum is omitted:

$$\boldsymbol{v} = \sum_{i=1}^{n} v^i \boldsymbol{e}_i = v^i \boldsymbol{e}_i \,. \tag{A.1}$$

~~The metric tensor $\boldsymbol{g}(\cdot,\cdot)$ is a bilinear form that takes in two vectors (from the same vector space) $\boldsymbol{u} = u^i \boldsymbol{e}_i$ and $\boldsymbol{v} = v^j \boldsymbol{e}_j$ and outputs a number:~~

$$\boldsymbol{g}(\boldsymbol{u},\boldsymbol{v}) = g_{ij} u^i v^j, \tag{A.2}$$

~~where $g_{ij} = \boldsymbol{e}_i \cdot \boldsymbol{e}_j$ are the covariant components of the metric tensor, which tell us about how the local geometry of a manifold changes. One can also think of the metric tensor as a "bar scale" at each point in a manifold, indicating how a local patch of geometry is stretched or squished and how measurements of length change. For example, consider the metric tensor for an n-dimensional Euclidean space (which is a flat space). Its metric tensor is one where all its diagonal components are 1 and the rest are 0: $g_{ij} = \mathrm{diag}(1,\ldots,1)$, where $i, j = 1, 2, \ldots, n$. The metric tensor for Minkowski space (flat 4D spacetime) is given by $\eta_{\mu\nu} = \mathrm{diag}(-1,1,1,1)$ or $\eta_{\mu\nu} = \mathrm{diag}\,(1,-1,-1,-1)$ depending on the convention used,~~

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Formatted: Bibliography, Space After: 12 pt where μ, ν = 0, 1, 2, 3. An index of 0 represents a time component and an index of 1, 2, or 3 represents a spatial component in the x, y, or z direction, respectively. This is the metric tensor for the local rest frame of an observer at a point (since an observer always appears at rest with respect to itself, which implies a locally flat spacetime). The inverse metric $\boldsymbol{g}^{-1}$ has contravariant components g^{ij} defined by

$$g^{ik} g_{kj} = \delta^{i}_{j}, \tag{A.3}$$

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where δ^{i}_{j} = diag(1,1,1,1) is the Kronecker delta (in 4D).

Author 100063 - Submission 100058 - Initial Submission Review

The paper has many strong features, including the clear question under consideration, the data provided to support the conclusions as well as the structure provided. Any reader would have an easy time reading the paper, since the paper nicely expands on the topic and includes detailed and well explained background theories. What is more, the paper's sections follow a logical order, and are all properly connected.

Even though there is no data analysis by the student itself, the paper supports its main claims with the current/existing literature. Many papers used throughout this work, are from recent studies, showing the up-to-date relevance of the paper. The paper helps summarising how the new observational data/constraints allow for new models of Dark Energy to be discussed/promoted.

The author would still benefit from some suggestions, even though the paper doesn't lack clarity. Suggestions (as listed below) are mostly related to figures and equations. Overall, the author seems confident with the level of mathematics used in the paper and makes a good use of the appendix in the end to explain mathematical notation not as common to high school students. Moreover, the author communicates clearly the ideas under consideration with the data provided backing up the author's claims.

When scanning the paper for plagiarism and AI, I get high percentages for AI detection (using turbo search). I suspect this could be a false positive, especially if the student used tools such as Grammarly. I would suggest that the team double checks this.

Finally, I would gladly accept this paper for publication with only minor revisions. The table below outlines suggestions that could help better this paper.

Line Number	**Quote from paper**	**Suggestions/Comments**	**Response from author**
9-14	We first give an overview of observational evidence for dark energy and a review of ΛCDM. Then…	The abstract is very good, but using passive voice sometimes helps with the tone of the paper. For example, instead of: we discuss the current issues with ΛCDM, you can say – current issues with ΛCDM are discussed etc.	I changed the wording of many sentences and removed "we"s.
24	undergoes accelerated expansion Riess et al., 1998),	Typo: (missing	Added (.
70	of state (EoSs)	Is it (Eos) instead?	Fixed to EoS.
154	, $\dot{} = \partial/\partial t$ denotes a derivative with respect..	Could be added in the appendix since (˙) is also mentioned earlier?	Added to appendix and deleted redundant sentence.
162-163	Here, we use the ([illegible] + + +) metric signature. If instead we were to..	The squares should change to (-)	Deleted sections detailing what the metric tensor is for simplicity.
179	the present-day value of H(a), called	Should it be H(t)?	Fixed.

212-213	density parameter for matter is Ωm ≈ 0.3 and that for dark energy is ΩΛ ≈ 0.7. The coincidence problem then asks the question of why Ωm and ΩΛ are of the same order of	Font error	***There doesn't appear to be any issues with font when I open the document at my end; the font error is likely an inconsistency in the text rendering in different applications. I will provide a PDF version to show that the font should and can render properly with the correct font packages. I use Lora, Cambria, and Cambria Math (for equations).
241-242	However, the potential *V* must in turn be fine-tuned, which shifts the coincidence problem to "why this potential?".	Can we expand as to why we then choose quintessence, if still fine-tuning is required?	Added.
253-255	Derivation of equations	Readers could benefit from some workings. (maybe in the appendix?)	Added additional workings in previous equation.
265	Any region of	Font error	***See response to 212-213.
273-274	2=d/dφ and V2	Does student mean $V^{'}$?	Fixed.
279	And α>0 is a constant	Could be another letter since (a) is used for cosmic scale factor and quite similar to (α).	Changed to β.
285	In the early stages and increase..	Font error	***See response to 212-213.
295-299	Equations	I believe it would have been better if $\widetilde{F}(a)$ switches order with F(α).	Order switched.
300	$\Omega_0 \approx 0.7$ is the present-day value of	Isn't it *Ω_Λ*? Also, font error.	Fixed symbol. For font error: ***See response to 212-213.
290 and 303	Figure 2 and 3.	Could add a subscript to w(a) to separate the freezing with the thawing potential.	Added.
325-326	only a linear parametrization (in *a*) and many quintessence potentials lead to a non-linear *w*	Nice to offer explanation for deviation.	I added examples of potentials to help elucidate (i.e. by specifying that the thawing and freezing quintessence potentials both

			lead to a non-linear *w*). However, providing a derivation for the EoS themselves will take up too much space (and not too relevant), hence I decided not to add them.
328	From figure 4..	I would have preferred to see subfigures such as 4a and 4b so that the author can point to them directly.	Added.
328	It is likely	Font error.	***See response to 212-213.
331	Its EoS is given by	Font error.	***See response to 212-213.
332	Equation 5.1	Can just mention equation 4.3	Equations 5.1 and 4.3 are different; *w* of phantom in 5.1 contains a negative kinetic energy component, so writing it out would probably be clearer to the readers.
385	Is still supported	Font error	***See response to 212-213.
387	HLQ is motivated through physical theory	Is there a paper for this? Can we see a reference?	Added citation.

A general suggestion would also be to number the pages of the paper and to avoid the # that indicates the equation number. For example:

$$a = \frac{1}{1+z}.\#(2.2)$$

Can be written as:

$$a = \frac{1}{1+z} \qquad (2.2)$$

Response to above suggestion: The formatting at my end appears normal (i.e. without the #). When I type #(equation number) and press return, the equation automatically formats with the equation centered and the equation number to the right.

Review of “A Fifth Fundamental Force? A Comparative Review Between ΛCDM and Quintessence”

This review paper, written by a high school student, is an impressive attempt to tackle one of the most complex problems in modern cosmology: the nature of dark energy. First and foremost, I want to recognize the ambition and effort involved here. Engaging with Einstein’s field equations, Friedmann models, quintessence potentials, and even the latest DESI observational data is far from trivial, and the author shows both enthusiasm and persistence in learning advanced material. For someone at the high school level, this is an outstanding achievement.

Strengths: The paper is well-structured and demonstrates a clear attempt to walk the reader through the history of dark energy research, from Einstein’s cosmological constant to modern dynamical models. The sections on observational evidence (supernovae and BAOs) are explained in a way that shows good understanding of the underlying concepts. The author also demonstrates awareness of current debates, such as the Hubble tension and the cosmological constant problem, which shows they are engaging with cutting-edge questions. The inclusion of equations and plots reflects a serious effort to connect theory and data, which is remarkable for a high school researcher.

Areas for Improvement: Because this is written at an early stage in the author’s academic journey, there are natural areas where improvement is needed. The biggest issue is that the paper sometimes feels more like a compilation of textbook material than a critical review. At this level, it is absolutely fine to rely heavily on summarizing existing literature, but the next step is to move from “describing” models to “evaluating” them — asking, for example, *why* quintessence may be more plausible than ΛCDM in practice, and *what the weaknesses* of each model are. The explanations of the freezing and thawing quintessence models, for example, are mathematically correct, but the physical meaning of these equations is not always made clear in plain language. A good exercise for revision would be to imagine explaining these models to another high school student: what is the simplest way to capture why they matter? Finally, the writing could be streamlined. At times the paper becomes long and dense, with many technical details that risk overwhelming readers. Trimming repetition and adding more plain-language explanations would make the argument more accessible and pedagogically stronger.

Recommendation: Overall, this paper shows excellent initiative and intellectual curiosity, and I want to strongly encourage the author to keep pursuing cosmology at higher levels of study. For publication, however, the paper would benefit from **major revisions**: clearer explanations in plain language, more critical evaluation of the models, and tighter organization. I would therefore recommend **“Accept with major revisions”.** With further work, this could become a very strong student contribution that not only summarizes technical material but also helps other learners understand the big picture of dark energy research.

Responses to areas for improvement:

- Added some detail to the explanations for the equations of state for thawing and freezing models, referencing the equation of motion and relating them to the behavior of the potential function through time. Also restructured/edited some of the wording to make it more streamlined/logical. I have further explained the motivation behind having subtypes of quintessence and the ideas behind each of them (i.e. how and why their potentials and EoS behave the ways they do).
- Added onto explanations for the weaknesses and plausibility of quintessence in Sections 4 and 6 (and made them clearer through changing the wording), comparing them to ΛCDM.
- Added a short plain-language explanation for the stress-energy tensor and how it is important for the EoS and predicting the universe’s evolution.
- Added a paragraph at the end of section 5 giving an overall analysis/conclusion of the results discussed, linking to the flaws of quintessence and quintom, stating that modified gravity models may be required if we were to completely resolve problems like Hubble tension or the cosmological constant problem.
- Removed repetition of the QFT/cosmological constant problem and the history behind discovering the universe’s acceleration in the introduction and Section 2.
- Removed the line element equation, fixed the 2nd FE and redshift equations, and removed sections on the inverse metric tensor in the Appendix and Section 4 when mentioning the mixed-component form of the stress-energy tensor.
- Reworded the captions for Figures 4 and 5 to reduce repetition.

Academic Integrity Guidelines: GPT Policy, Student & Parent Honor Code, Mentor Honor Code
Indigo Contact Information: hello@indigoresearch.org
Indigo Research IRIS Webpage

Dear Convergence,

We write in reference to [redacted] who we taught through Indigo Research's 2025 IRIS Summer Program.

[redacted] performance in our Astrophysics course gave us every reason to believe that his final paper is an original work of scholarship. After watching [redacted] develop his paper over the course of our six-week seminar, meeting with him individually, and reading his final paper, it is of our opinion that this paper should be allowed to go through the peer-review process as independent work.

For further screening against AI we ran his final paper through the Grammarly AI plagiarism check and only flagged 8% of the texts as *potential* AI-generated text.

While it is impossible to confirm with absolute certainty that no AI was involved in the writing of this paper, we write as an instructor in support of [redacted] academic integrity. We are sending this letter to ensure the usage of LLMs and plagiarism concerns are alleviated, so that doesn't prevent [redacted] excellent paper from publication.

Sincerely,

Dr. Chima McGruder (instructor)
c.mcgruder@indigoresearch.org

Catherine Petretti (teaching assistant)
c.peretti@indigoresearch.org

Astrophysics
IRIS Program 2025

In the initial stages of research and planning, I used ChatGPT to clarify basic astrophysical terminology and concepts, as I was relatively new to the topic. For instance, I asked ChatGPT what baryon acoustic oscillations are, which cosmological problems different dark energy models solve, and whether there is a hypothetical singular “correct equation of state”. I also asked it to explain certain mathematical expressions I was less familiar with. After every request to ChatGPT I made about clarifications and explanations, I double checked using Google Scholar for papers and Bing for websites. I also used ChatGPT to help me understand complex plots and statistical terminology (like what a posterior distribution is) within papers. I asked ChatGPT for suggestions of dark energy models to analyse and how I can analyse/compare them (e.g. via equation of state, Friedmann equations, running simulations? etc.). I also asked my mentor and teaching assistant for advice and suggestions on the main models and aspects to focus on, as well as how I could structure my paper.

I also initially planned to run simulations given the equation of state and Friedman equations to compare with data, which required astropy and other bits of software, so I asked ChatGPT for help on installing the required packages and setting up for the simulation (as well as from online documentation). However, I later decided to use data available online only and not run simulations because the process of learning took too long (I was relatively new to coding for simulations), and technical problems occurred. I created every relevant TikZ plot myself (and properly cited figures which I took from other papers) with no help from AI and purely from online documentation.

In the process of writing my first draft, I used Grammarly and Word’s grammar and spelling checker, although after realising that I was not supposed to use Grammarly in the process until after finishing my full draft, I reverted my paper back to the previous version with no Grammarly assistance. After finishing my final full draft, I used Grammarly and ChatGPT to check grammatical and spelling mistakes as well as improvements to wording/phrasing. I did not use Grammarly to rewrite entire sections and then paste them, nor did I use AI to paraphrase from other papers; I asked ChatGPT for clarification on any words or phrases I didn’t know so I could better understand their ideas, which I then paraphrased from my own understanding. I used Zotero for citing sources when paraphrasing.

Final decision on "A Fifth Fundamental Force? A Comparative Review Between ΛCDM and Quintessence"

I believe the student put in some honest work and considered our recommendations. The student not only fixed typos and notation issues but made sure to explain and expand on some parts of the paper such as why scientists choose quintessence. I would happily accept this work for publication as it offers a high-standard paper for a high school student.